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Performance Analysis of Different Inverse Filter Design Techniques

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Abstract- ***In modern communication technology design of inverse filter i.e. channel equalizer on receiver side is important in order to combat the effects of channel distortion. In this paper we address different approaches of inverse filter design. The different approaches used are based on second order statistics methods such as Shank's algorithm, Wiener- Hopf equations (both FIR and IIR), LMS algorithm. We have have modeled communication channel as FIR filter as most of the communication channels can be modeled as FIR channel and followed above mentioned approaches to design an inverse filter in order to get back best estimate of original source signal. Simulation results show performance of these with respect to computational complexity, inverse filter order and SNR vs. BER plots. Above effects have been observed for different channel distortion values.***

Keywords- Inverse filter, Channel equalization, least square method, Wiener-Hopf equations, Wiener IIR filters, LMS algorithm.

I. INTRODUCTION

Inverse filter plays important role in different communication and signal processing applications such as seismic signal processing. In a digital communication system for example a signal is to be transmitted across a non-ideal channel. Assuming that channel is linear and has system function H(z) to minimize chance of making errors we would like to design a channel equalization filter whose frequency response is exact inverse (ideally) or approximate inverse of channel H(z). Thus our aim is to find an equalizer or inverse filter H_{inv}(z) such that

$$H(z)H_{inv}(z) = 1$$
or
$$H(n) * H_{inv}(n) = \delta(n)$$

In most of the practical cases inverse system $H_{inv}(z) = 1/H(z)$ is not a perfect solution. This is because we should have H(z) to be minimum phase then only it is possible to get inverse filter both causal and stable otherwise inverse filter will be non-causal and/or unstable. Another limitation with the above solution of inverse filtering is that in some applications, it may be necessary that H_{inv}(z) should be FIR filter, so inverse filter will be infinite in length unless h(n) is all pole filter [1]. Another practical limitation with above solution is if due to fading effects, multipath effects in the practical communication channel there may be possibility of spectral nulls at particular frequency so inverse filter design by above solution will produce very high gain at that particular frequency and this leads to noise amplification if signal to noise ratio (SNR) is low and also complex detection mechanism at receiver.

When channel is ideal then only it is possible to use classical filters such as low pass, high pass, band pass etc. to restore the desired original signal. But in practical cases it is necessary to use optimum filters that will produce best estimate of desired signal, Optimum filters include wiener filters (digital), Kalman filters (discrete). Wiener filters consider the problem of designing filter that will produce MMSE estimate of desired signal [1].

While designing equalizers in communication systems to compensate channel effects it is necessary that equalizer should track changes in communication channel. For this reason there is need of adaptive algorithm such as LMS algorithm that will update equalizer coefficients [6].

In this paper we will demonstrate FIR least square inverse filtering problem based on Shank's method, optimum inverse filtering approach both for FIR, IIR filters and minimization of mean square error (MMSE) approach for inverse filter design using LMS algorithm.We will also compare different aspects of these methods.

II. SYSTEM OVERVIEW AND FILTER MODEL

In this section we are going to define system model that defines relationship between received signal x (n) and desired signal d (n) as:

x (n) = d(n) * h(n) + v(n) (1)

Where v (n) is additive white Gaussian noise with zero mean and variance σ_v^2 = 0.001.

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In this paper for simulation purpose channel model taken is as follows:

In practical communication systems most of the channels can be modeled as FIR filters, as these are practically realizable filters. Here we have considered communication channel as shown below [2]

$$h(n) = \begin{cases} 0.5\left(1 + cos\dfrac{2\pi(n-2)}{W}\right) & n = 1,2,3 \\ 0 & \end{cases}$$

Where; W controls amount of amplitude distortion introduced by channel. The coefficients of this FIR filter are [2]:

$$\mathrm{h(n)} = \{0, 0.5(1+\cos\tfrac{2\pi}{\mathrm{W}}), 1, 0.5, (1+\cos\tfrac{2\pi}{\mathrm{W}}), 0\}$$

This is symmetric channel.

III. PROBLEM FORMULATION

A. Design of least square inverse filters (Shank's method)

Inverse filter design problem can be formulated as follows [5]:

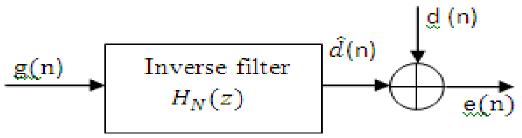

Fig. 1: Model of inverse filter for shank's method

From figure (1),

$$\text{e (n)} = \text{d(n)-} \hat{d}\text{(n)} \quad (2)$$

$$\text{e (n)} = \text{d(n)-h(n)*g(n)} \quad (3)$$

Using least square approach in order to minimize sum of squares is given as:

$$\zeta_{min} = \sum_{n=0}^{\infty} |e(n)|^2 \quad (4)$$

$$\zeta_{min} = \sum_{n=0}^{\infty} |d(n) - \sum_{l=0}^{N-1} h_N(l) g(n-l)|^2 \quad (5)$$

Solution for optimum least square inverse filter is given as:

$$\sum_{\mathrm{l}=0}^{\mathrm{N}-1} \mathrm{h_N(l) r_g(k-l)} = \mathrm{r_{dg}(k)} \quad (6)$$

Here k = 0, 1, 2….N-1, r_{dg} is cross-correlation between desired signal d (n) and input signal to inverse filter g (n) and is given by:

$$\mathrm{r_{dg}(k)} = \sum_{\mathrm{n}=0}^{\infty} \mathrm{d(n)\, g^*(n-k)} \quad (7)$$

In matrix notation equation (5) becomes:

$$\begin{bmatrix} r_g(0) & r_g^*(1) & \cdots & r_g^*(N-1) \\ \vdots & & \ddots & \vdots \\ r_g(N-1) & r_g(N-2) & \cdots & r_g(0) \end{bmatrix} \begin{bmatrix} h_N(0) \\ \vdots \\ h_N(N-1) \end{bmatrix} = \begin{bmatrix} r_{dg}(0) \\ \vdots \\ r_{dg}(N-1) \end{bmatrix} \quad (8)$$

$$\mathrm{R_g h_N} = \mathrm{r_{dg}} \quad (9)$$

Where R_g is autocorrelation matrix of input signal to inverse filter h_N is inverse filter coefficient matrix, so:

$$\mathrm{h_N} = \mathrm{r_{dg}}.R_g^{-1} \quad (10)$$

Minimum mean square is given as in terms of filter coefficients as [1]:

$$\zeta_{\min} = \mathrm{r_d(0)} - \sum_{\mathrm{k}=0}^{\mathrm{N}-1} \mathrm{h_N(k) r_{dg}^*(k)} \quad (11)$$

B. Inverse filter design based on FIR Wiener filter method

In Wiener filtering problem, design of filter to recover a signal d (n) from noisy output of channel

$$\text{x (n)=d(n)+v(n)} \quad (12)$$

Where v (n) is noise introduced in channel.

Wiener filtering problem can be modeled same as FIR least square inverse filtering problem only input to inverse filter changes from g (n) to x (n) i.e. noisy input.

C. Wiener Inverse IIR filters design

Unlike the case of FIR Wiener filters where only finite filter coefficients are to be determined, in case of Wiener IIR filters there are infinite number of filter coefficients are to be determined. In this method we will consider two approaches [4]:

1. Wiener IIR Non-causal, where we will not place any constraints on solution.

2. Wiener IIR Causal, where we constrain solution to be causal by forcing filter coefficients to be zero for n≤0.

C.1 Wiener IIR Non-Causal filter

For IIR Wiener filter in order to find unit impulse response h(n) from IIR filter given by [4]

$$H(z) = \sum_{n=-\infty}^{\infty} h(n)\, Z^{-n} \qquad (13)$$

As IIR Wiener filter is Non-Causal.
Wiener-Hopf equations for Non-Causal IIR filters are given by

$$\sum_{l=-\infty}^{\infty} \mathrm{h(l) r_x(k-l) = r_{dx}} \quad ; -\infty < \mathrm{k} < \infty \qquad (14)$$

Comparing above equations with FIR wiener the only difference is in limits of summation values.
The solution to above equations can be directly written in terms of convolution as

$$\mathrm{h(k) * r_x(k) = r_{dx}(k)} \qquad (15)$$

Above equation in frequency domain given as

$$\mathrm{H(e^{j\omega})P_x(e^{j\omega}) = P_{dx}(e^{j\omega})} \qquad (16)$$

Where $P_{dx}(e^{j\omega})$ is cross power spectral density between desired signal $d(n)$ and received signal $x(n)$.Therefore frequency response of IIR filter is

$$\mathrm{H(e^{j\omega}) = \frac{P_{dx}(e^{j\omega})}{P_x(e^{j\omega})}} \qquad (17)$$

Mean square error for IIR Wiener is given as [4]

$$\zeta_{\mathrm{min}} = \mathrm{r_d}(0) - \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{H(e^{j\omega})P_{dx}^*(e^{j\omega})\ d\omega} \qquad (18)$$

$$\text{Where,} r_d(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} P_d(e^{j\omega}) d\omega \qquad (19)$$

C.2 Wiener IIR Causal filter

As mentioned earlier due to change in limits in case of Causal case it is not possible to express r_{dx}(k) as convolution sum of h (k) and r_x(k).
To solve Wiener-Hopf equations as a special case we need to give unit variance white noise $\varepsilon(n)$ as input to filter

$$\sum_{l=0}^{\infty} g(l)\, r_\varepsilon(k-l) = r_{d\varepsilon}(k)\,; \quad 0 \le k < \infty \qquad (20)$$

Where g(n)=$r_{d\varepsilon}(n)u(n)$ for causal case with noisy input $\varepsilon(n)$. g(n) can be expressed in Z-domain as

$$\mathrm{G(z) = [P_{d\varepsilon}(z)]_+} \qquad (21)$$

"+" indicates 'positive time part' of sequence whose z-transform is contained in bracket. If x(n) is random process then it's spectral factorization is given as

$$\mathrm{P_x(z) = \sigma_0^2 Q(z) Q^*(1/z^*)} \qquad (22)$$

Where Q (z) is minimum phase transfer function.
The desired solution of IIR inverse filter is given as [4]

$$H(z) = \frac{1}{\sigma_0^2 Q(z)}\left[\frac{P_{dx}(z)}{Q^*(1/z^*)}\right]_+ \qquad (23)$$

Mean square error is given by

$$\zeta_{min} = \frac{1}{2\pi}\int_{-\pi}^{\pi}[P_d(e^{j\omega}) - H(e^{j\omega})P_{dx}^*(e^{j\omega})]d\omega \qquad (24)$$

In time domain, above equation can also be written as

$$\zeta_{min} = r_d(0) - \sum_{l=0}^{\infty} h(l) r_{dx}^*(l) \qquad (25)$$

D. Design of inverse filter using MMSE approach (using LMS algorithm)

For adaptive filtering Wiener-Hopf equation cannot be used because:

1. It requires knowledge of autocorrelation $r_x(k)$ and crosscorrelation $r_{dx}(k)$ which are generally unknown.

2. If Toeplitz matrix R_x is ill-conditioned (almost singular) then solution given by Wiener-Hopf equation is numerically sensitive to round-off error, finite precision effects.

In many practical aspects channel is time varying so design of time varying (adaptive) inverse filter at receiver is much more complex than time invariant filter. This is because filter coefficients are to be updated at each time when channel is varying. Filter coefficient update equation is given as [1]:

$$\mathrm{W_{n+1} = W_n + \Delta W_n} \qquad (26)$$

Where correction ΔW_n is applied to filter coefficients W_n , at time 'n' to get new set of coefficients,W_{n+1}, at time $n+1$.

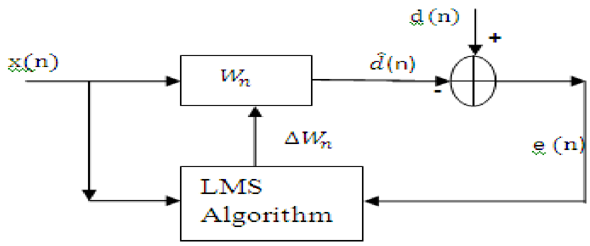

Fig. 2: Adaptive weight updating mechanism in LMS algorithm

In this paper, we have considered stationary channel. In a stationary environment, adaptive filter should produce a sequence of correction ΔW_n in such a way that W_n converges to a solution to Wiener-Hopf equation [7]

$$\lim_{n\to\infty} W_n = R_x^{-1}.r_{dx} \tag{27}$$

The LMS Algorithm

LMS algorithm used in FIR adaptive filters and it is extension of steepest descent algorithm. The weight update equation is given by [1]:

$$\mathrm{W_{n+1} = W_n + \mu e(n) x^*(n)} \tag{28}$$

Where μ is step size. Error e(n) can be calculated as:

$$\mathrm{e(n) = d(n) - y(n)} \tag{29}$$

Where d (n) is desired signal and y(n) is output of equalizer. Estimated output y (n) can be evaluated as:

$$\mathrm{y(n) = W_n^T x(n)} \tag{30}$$

Where W_n^T is inverse filter weights vector and x(n) is input to inverse filter.
For jointly wide sense stationary processes, d(n) and x(n), LMS algorithm converges in the mean if [1]

$$0 < \mu < \frac{2}{\lambda_{\max}} \tag{31}$$

Where λ_{max} is largest Eigen value of R_x .Above equation has following limitations:

1. Upper bound is too large to ensure stability of LMS algorithm.

2. As equation converges with mean it does not give clear idea about variance of W_n.

More sophisticated upper bound for step size is given by:

$$0 < \mu < \frac{2}{(p+1)E\{|x(n)|^2\}} \tag{32}$$

Where 'p' is order of inverse filter.

IV. SIMULATION RESULTS

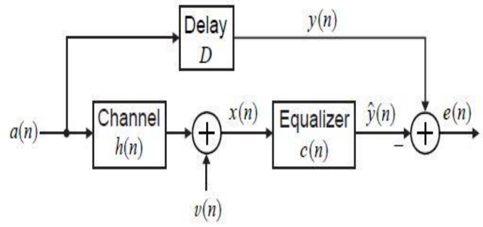

Fig. 3 Frequency response of assumed communication channel.

A. Shank's Method

Shank's method assumes ideal channel so no noise considerations are involved. Fig. 4 shows frequency response of least square inverse filter by using Shank's method. Below response illustrates that inverse filter has frequency response exact opposite to channel or filter, so it minimizes distortions introduced in channel.

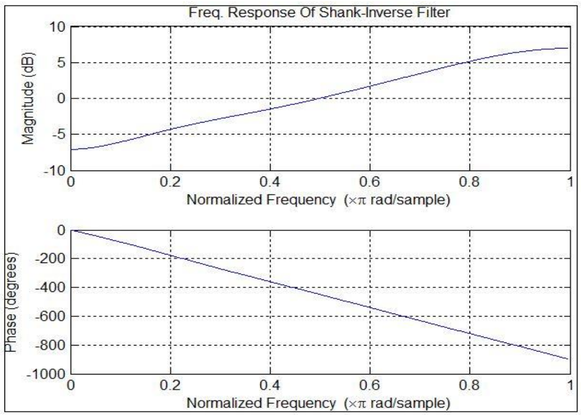

Fig. 4: Frequency response of least square inverse filter.

Fig.5 shows that an application of Shank's method, in design of least square inverse filter, coefficients of inverse filter are found out such that error between desired signal and estimated output is almost zero.

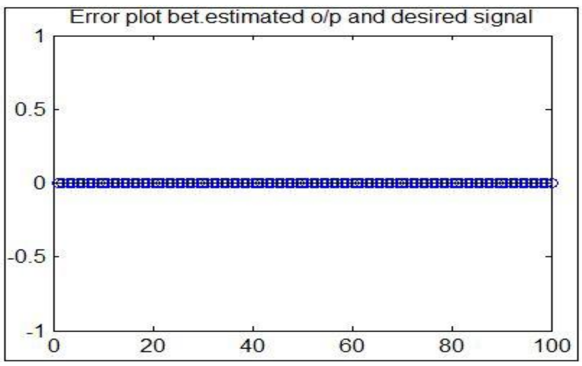

Fig. 5: Error between estimated and desired output

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B. FIR – Wiener filter method

Fig. 6 shows frequency response of Wiener equalizer. Wiener filtering methods considers problem when input data to inverse filter is corrupted with noise.

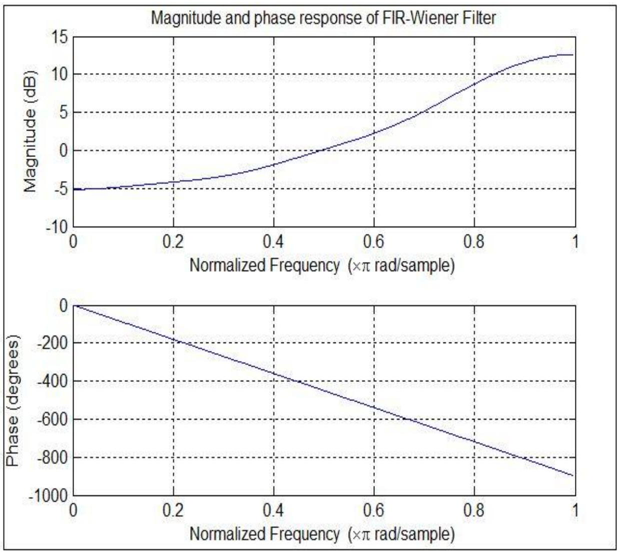

Fig. 6: Frequency response of FIR-Wiener inverse filter.

Fig. 7 demonstrate how BER changes as with SNR for different conditions of channel distortion (w controls distortion introduced in channel). As w increases from 2.9 to 3.5 BER also increases and settles to zero for larger values of SNR, which is evident from figure.

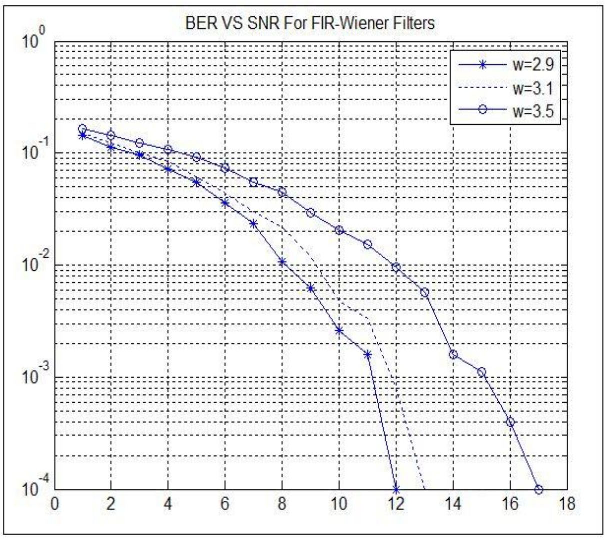

Fig 7: SNR Vs BER at w=2.9, 3.1 and 3.5 for FIR- Wiener inverse filter.

C.1. IIR Wiener non-causal filter method

Fig. 8 and 9 gives frequency response of estimated inverse filter. In this case order of inverse filter that gives nearly inverse response as that of channel is found to be less unlike the case of FIR Wiener filters.

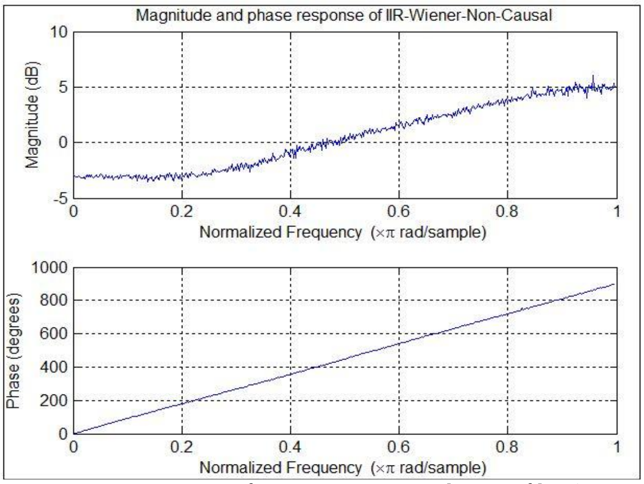

Fig. 8: Frequency response of IIR Wiener non-causal inverse filter (inverse filter order=11)

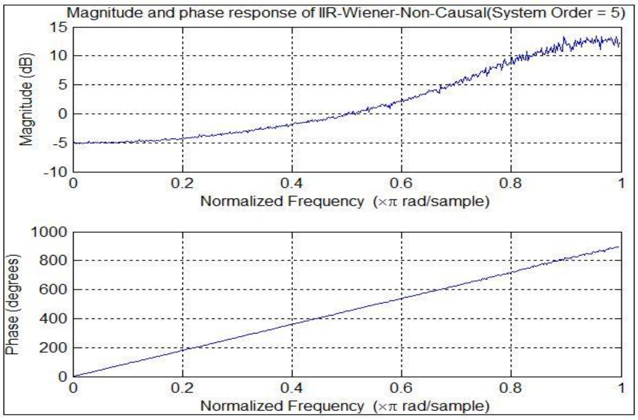

Fig. 9: Frequency response of IIR Wiener non-causal inverse filter (inverse filter order=5)

From the below figure, it can be noted that as distortion introduced in channel increases IIR Wiener non-causal filter fails to reproduce the desired signal for all signal values that is BER approximately remains constant , even if increase in SNR values.

In comparison to FIR Wiener Filters BER decreases gradually for w=2.9 and settles down to a minimum value (but not zero) later.

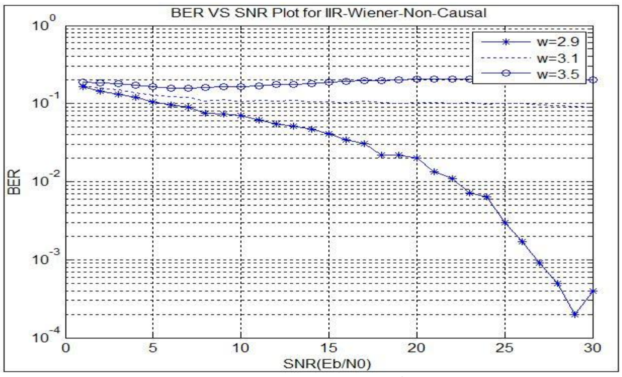

Fig. 10: SNR Vs BER at w=2.9, 3.1 and 3.5 for IIR Wiener Non-causal inverse filter.

C.2. IIR Wiener causal filter method

Fig. 11 shows inverse filter frequency response which is nearly inverse but there is change in magnitude response values this is because we are placing constraint on filter that it should be causal.

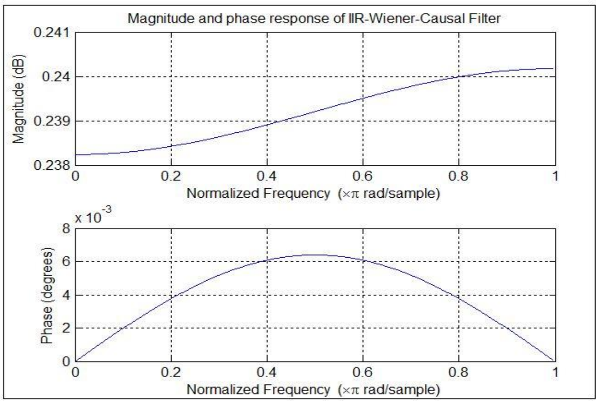

Fig 11: Frequency response of II
R on-causal inverse filter (inverse filter order=11)

D. MMSE –LMS method

Below figure shows inverse filter frequency response for MMSE-LMS filter. This method is works on the principle of adaptive updating of filter coefficient based on channel so it is best solution than Wiener filtering methods.

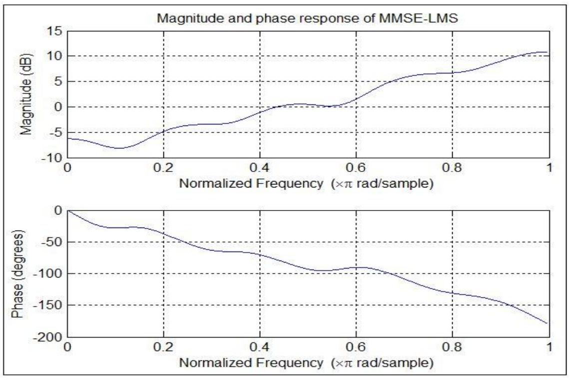

Fig. 12: Frequency response of MMSE-LMS inverse filter.

Fig. 13 shows plot for SNR Vs BER for different values of channel distortion (w). As w increases with same step size BER decreases gradually as SNR increases.

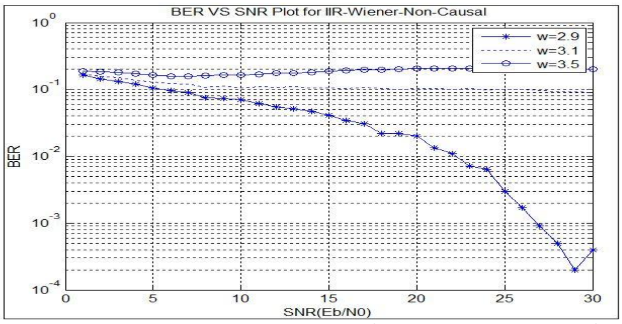

Fig.13: SNR Vs BER at w=2.9,3.1 and 3.5 for MMSE-LMS inverse filter.

Below figure demonstrates effect of step size on BER. From eq.—we got step size in between 0<μ<0.16 .So if we take value of step size greater than 0.16 (0.17 here), we got nearly constant BER.

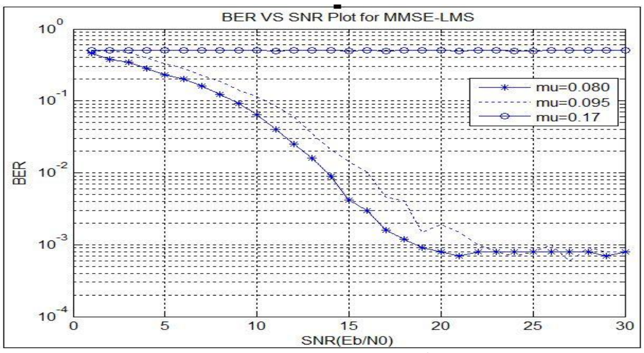

Fig. 14: SNR Vs BER at μ=0.08,0.095 and 0.17 for MMSE-LMS inverse filter.

Below fig. 15 shows plots for different values of μ.If we take step size very close to lower bound of equation (32) (μ=0.0001) then equalizer settles down very slowly and MSE remains approximately constant. If we take μ=0.095, which is in between bounds of equation (32), after some iterations MSE nearly approaches to zero and equalizer freezes it's coefficients until change in the channel. If we take μ=0.17, which is greater than upper bound of equation (32), MSE is very high.

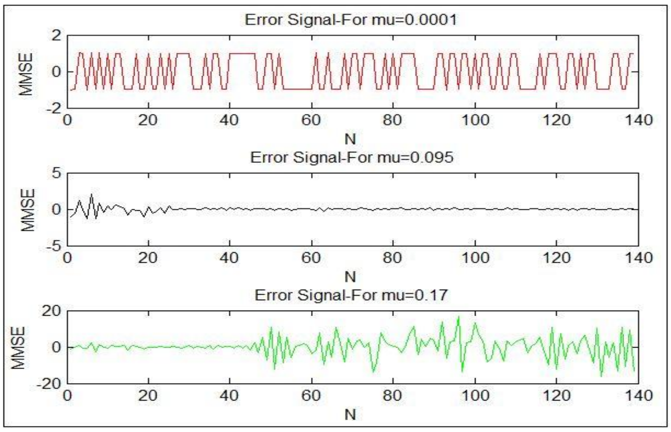

Fig. 15: Error signal for three different step sizes (μ=0.0001, 0.095&0.17)

TABLE I

Minimum Least Square error for different methods

Method name:	Shank	FIR Wiener	IIR-Noncausal	IIR-causal
ζ_{min}	0.0416	0.1826	-0.6126	0.8876

In Table I, the modeling error for different methods is given. It can be concluded that as we move from shank's method to FIR-wiener the error is increased due to introduction of channel noise in FIR-Wiener filters. Whereas for IIR-Noncausal filters the error is minimum (desired) but problem of Non causality's can be solved by placing some constraints on filter to make it causal which again leads to high error(undesirable).

V. CONCLUSIONS

In this paper, different channel equalization approaches have been presented in order to minimize effects of channel distortion. In all the approaches inverse filter frequency response is observed and are found to be nearly opposite to that of channel. In first approach, it is observed that error between estimated output and desired signal is zero ideal channels (in absence of noise). In preceding approaches, we have observed SNR Vs BER plot for different values of channel distortion parameter (w). In FIR-Wiener case it is found that for large value of w, BER gradually approaches to zero as SNR increases, so FIR Wiener filters performs better under noisy environment. In IIR Non-causal case, BER for higher value of w is nearly constant, so we can conclude that IIR filters are inefficient when channel is noisier. In MMSE-LMS approach, with same step size for high value of w BER decreases gradually which means LMS algorithm will not perform better if distortion is more and step size is constant. On the other hand if step size is varied within the bounds then LMS algorithm perform better under noisy environment. If we change the step size beyond upper bound then BER nearly remains constant and MSE increase rapidly. So we can say that as step size increases LMS algorithm will converge fast but MSE increases.

REFERENCES

i. *Monson H. Hayes, "Statistical digital signal processing and modelling", john Wiley and sons,2004.*

ii. *Manolakis, Ingle, Kogon, "Stastical and Adaptive signal processing" Artech Housepublications,2005.*

iii. *J.G.Proakis, "Digital communications", Tata Mc Graw Hill publication, 4th edition, 2001.*

iv. *Chun-yan Gu, Hong-feng Yi, Ke-chu Yi, Li-jun Jin, "A Method of Inverse Filter Design Based on Cepstrum Measure" IEEE 19th international conference on advanced information and networking applications 2005.*

v. *Shih Yu Chang, Hasio-Chun Wu "New FIR filter design for both spectrum matching and inverse system approximation"IEEE Globecomm proceedings 2010.*

vi. *An efficient Recursive Inverse adaptive filtering algorithm for channel equalization,Ahmad, M.S.; Kukrer, O.; Hocanin A.,IEEE Wireless Conference (EW), 2010*

vii. *Wiener-Hopf equations techniques for generalized variational inequalities and fixed point problem,Image and Signal Processing (CISP), 2011 4th International Congress on,2011*