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Determining Economic Production Quantity in the Presence of Varying Item Size and Stochastic Demand

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Abstract : The classical Economic Production Quantity (EPQ) model as applied in production-inventory management assumes constant demand of a single item, often of a specific size. In this paper, an optimization method is developed for determining the EPQ of an item with varying size and stochastic demand. Adopting a Markov decision process approach, the states of a Markov chain represent possible states of demand for a given size of item. The decision of whether or not to produce additional units of a specific size of item is made using dynamic programming over a finite period planning horizon. Empirical results show the existence of an optimal state dependent economic production quantity for a specific size of item as well as the corresponding production-inventory costs.

Keywords: *Economic Production Quantity, Item size, stochastic demand*

1. Introduction

In this era of industrial competitiveness, the zeal for manufacturing industries to optimally establish cost-effective production-inventory levels of items is paramount. This is however a considerable challenge when the item produced for stock varies in size with demand uncertainty. Two major problems are usually encountered: (i) determining the most desirable period during which to produce additional units of a specific size of item in question and (ii) determining the economic production quantity (EPQ) of a specific size of item given a periodic review production-inventory system when demand is uncertain.

In this paper, a production-inventory system is considered whose goal is to optimize the economic production quantity and total production-inventory costs associated with producing and holding inventory. The item under consideration may be produced in three specific sizes; namely: small, medium or large. At the beginning of each period, a major decision has to be made: whether to produce additional units of a specific size of item in inventory or postpone production and utilize the available units in inventory.

According to Goya[1],the EPQ may be computed each time a product is scheduled for production when all products are produced on a single machine. The distribution of lot size can be computed when the demand for each product is a stochastic variable with a known distribution. The distribution is independent for non overlapping time periods and identical for equal time epochs. In related work by Khouja[2],the EPQ is shown to be determined under conditions of increasing demand, shortages and partial backlogging. In this case the unit production cost is a function of the production process and the quality of the production process deteriorates with increased production rate.Tabucanno and Mario [3] also presented a production order quantity model with stochastic demand for a chocolate milk manufacturer. In this paper, production planning is made complex by the stochastic nature of demand for the manufacturer's product. An analysis of production planning via dynamic programming approach shows that the company's production order variable is convex. In the article presented by Kampf and Kochel[4], the stochastic lot sizing problem is examined where the cost of waiting and lost demand is taken into consideration. In this model, production planning is made possible by making a trade-off between the cost of waiting and the lost demand.

The four model presented have some interesting insights regarding economic production quantity in terms of optimization and demand stochasticity.However,the concept of varying item size is not a salient factor in the optimization models cited.

The paper is organized as follows: After describing the mathematical model in §2, consideration is given to the process of estimating model parameters. The model is solved in §3 and applied to a special case study in §4. Some final remarks lastly follow in §5.

2. Model Formulation

An item is considered with varying size in a given production-inventory system whose demand during a chosen period over a fixed planning horizon is classified as either *Favorable* (state F) or *Unfavorable* (state U) and the demand of any such period is assumed to depend on the demand of the preceding period. The transition probabilities over the planning horizon from one demand state to another may be described by means on a Markov chain. Suppose one is interested in determining the optimal course of action, namely to produce additional units of a specific size of item (a decision denoted by K=1) or not to produce additional units (a decision denoted by K=0) during each time period over the planning horizon where K is a binary decision variable. Optimality is defined such that the lowest expected production-inventory costs are accumulated at the end of N consecutive time periods spanning the planning horizon for a given size of item. In this paper, a three-size(S=3) item for a two-period (N=2) planning horizon is considered.

2.1 Assumptions and Notation

Varying demand is modeled by means of a Markov chain with state transition matrix $Q^K(S)$ where the entry $Q^K{}_{ij}(S)$ in row i and column j of the transition matrix denotes the probability of a transition in demand from state iε {F,U}to state jε {U ,F}for item size S= {1,2,3}under a given production lot sizing policy

Kε {0,1}.The number of customers observed in the system and the number units demanded during such a transition is captured by the *customer matrix* $N^K(S)$ and *demand matrix* $D^K(S)$ respectively.

Furthermore, denote the number of units in inventory and the total (production, holding and shortage) cost during such a transition by the *inventory matrix* $I^K(S)$ and cost matrix $C^K(S)$ respectively. Also denote the *expected future cost*, the already *accumulated total cost* at the end of period *n* when demand is in state iε {F,U}for a given production lot sizing policy Kε {0,1}by respectively $e^K(S)$and $a^K_i(S,n)$and let $e^K(S)=[e^K_F(S),e^K_U(S)]^T$ and $a^K_i(S,n)=[a^K_F(S,n),a^K_U(S,n)]^T$ where "T" denotes matrix transposition.

2.2 Finite dynamic programming formulation

Recalling that the demand can be in state F or in state U, the problem of finding an optimal EPQ for size S of item may be expressed as a finite period dynamic programming model.

Let $C_n(i,S)$ denote the expected total production-inventory costs for size S of item accumulated during periods *n,n+1,........,N*given that the state of the system at the beginning of period *n* is iε {F,U}.The recursive equation relation C_n and C_{n+1} is

$$C_n(i,S) = min_K[Q^K_{iF}(S)(C^K_{iF}(S) + C_{n+1}(F,S), C^K_{iU}(S) + C_{n+1}(U,S))]$$

iε {F,U} ,S={1,2,3} ,*n=1,2,*................................*N* (1)

together with the final conditions

$$C_{N+1}(F,S) = C_{N+1}(U,S) = 0$$

This recursive relationship maybe justified by noting that the cumulative total production-inventory costs $C^K_{ij}(S)+C_{n+1}(j,S)$ resulting from reaching jε{F,U }at the start of period *n+1* from state iε{$\mathcal{F},\mathcal{U}$}at the start of period *n* occurs with probability $Q^K_{ij}(S)$ and hence the dynamic programming recursive equations

$$C_n(i,S) = min_K[e^K_i(S) + Q^K_{iF}(S)C_{n+1}(F,S) + Q^K_{iU}(S)C_{n+1}(U,S)]$$

iε{F,U} n=1,2,..........N S={1,2,3} Kε {0,1}. (2)

$$C_N(i,S) = min_K \ e^K_i(S) \quad (3)$$

result where (3) represents the Markov chain stable state.

2.2.1 Computing $Q^K(S)$,$C^K(S)$ and $P^K(S)$

The demand transition probability from state i ε{ F, U}to state jε{ F, U } given production lot sizing policy Kε {0,1} for size S of item may be taken as the number of customers observed when demand is initially in state i and later with demand changing to state j, divided by the number of customers over all states.

That is $Q^K_{ij}(S) = N^K_{ij}(S)/[N^K_{iF}(S) + N^K_{iU}(S)]$ (4)

iε{F,U} , S={1,2,3} Kε {0,1}

When demand outweighs on-hand inventory, the cost matrix $C^K(S)$ may be computed by the relation:

$$C^K(S) = [c_p(S) + c_h(S) + c_g(S)][D^K(S) - I^K(S)]$$

where $c_p(S)$ denotes the unit production cost, $c_h(S)$ denotes the unit holding cost and $c_g(S)$ denotes the unit shortage cost.

$$C^K_{ij}(S) = \begin{cases} [c_p(S) + c_h(S) + c_g(S)][D^K_{ij}(S) - I^K_{ij}(S)] & if \quad D^K_{ij}(S) > I^K_{ij}(S) \\ c_h(S)[I^K_{ij}(S) - D^K_{ij}(S)] & if \quad D^K_{ij}(S) \le I^K_{ij}(S) \end{cases}$$

for all i,j ε{F,U}, Kε {0,1} , S={1,2,3} (5)

A justification for expression (5) is that $D^K_{ij}(S) - I^K_{ij}(S)$ units must be produced in order to meet the excess demand. Otherwise production is cancelled when demand is less than or equal to on-hand inventory. The EPQ when demand is initially in state i ε{F,U}given production lot sizing policy K ε {0,1} is

$$p^K_i(S) = [D^K_{iF}(S) - I^K_{iF}(S)] + [D^K_{iU}(S) - I^K_{iU}(S)] \quad (6)$$

i ε{F,U}, Kε {0,1} , S={1,2,3}

The following conditions must however hold:

1. $p^K_i(S) > 0$ when $D^K_{ij}(S) > I^K_{ij}(S)$ and $p^K_i(S) = 0$ when $D^K_{ij}(S) \le I^K_{ij}(S)$
2. K=1 when $c_p(S) > 0$ and K=0 when $c_p(S) = 0$
3. $c_g(S) > 0$ when shortages are allowed and $c_g(S) = 0$ when shortages are not allowed.

3. Optimization

The optimal EPQ and production lot sizing policy are found in this section for each time period separately,

3.1 Optimization during period 1

When demand is Favourable (i.e. in state F), the optimal production lot sizing policy and production-inventory costs during period 1 are

$$K = \begin{cases} 1 & if \quad e^1_F(S) < \ e^0_F(S) \\ 0 & if \quad e^1_F(S) \ge \ e^0_F(S) \end{cases}$$

$$C_1(F,S) = \begin{cases} e^1_F(S) & if \quad K = 1 \\ e^0_F(S) & if \quad K = 0 \end{cases}$$

The associated EPQ is then

$$P^K_F(S) = \begin{cases} [D^1_{FF}(S) - I^1_{FF}(S)] + [D^1_{FU}(S) - I^1_{FU}(S)] & if \quad K = 1 \\ 0 \quad if \quad K = 0 \end{cases}$$

Similarly, when demand is Unfavorable (i.e. in state U), the optimal production lot sizing policy and production-inventory costs during period 1are

$$K = \begin{cases} 1 & if \quad e^1_U(S) < e^0_U(S) \\ 0 & if \quad e^1_U(S) \ge e^0_U(S) \end{cases}$$

$$C_1(U,S) = \begin{cases} e^1_U(S) & if \quad K = 1 \\ e^0_U(S) & if \quad K = 0 \end{cases}$$

In this case, the associated EPQ is

$$P^1_U(S) = \begin{cases} [D^1_{UF}(S) - I^1_{UF}(S)] + [D^1_{UU}(S) - I^1_{UU}(S)] & if \quad K = 1 \\ 0 \quad if \quad K = 0 \end{cases}$$

Using (2) ,(3) and recalling that $a^K_i(S,2)$ denotes the already accumulated total production-inventory costs at the end of period 1 as a result of decisions made during that period, it follows that

$$a_i^K(S,2) = e_i^K(S) + Q_{iF}^K(S)min[e_F^1(S), e_F^0(S)] + Q_{iU}^K(S)min[e_U^1(S), e_U^0(S)]$$
$$= e_i^K(S) + Q_{iF}^K(S)C_1(F,S) + Q_{iU}^K(S)C_1(U,S)$$

3.2 Optimization during period 2

Using dynamic programming recursive equation (1), and recalling that $a^K_i(S,2)$ denotes the already accumulated total cost for size S of item at the end of period 1 as a result of decisions made during that period,when demand is Favorable(i.e. in state F), the optimal production lot sizing policy and associated production-inventory costs during period 2 are

$$K = \begin{cases} 1 & if \quad a_F^1(S) < a_F^0(S) \\ 0 & if \quad a_F^1(S) \geq a_F^0(S) \end{cases}$$

$$C_2(F,S) = \begin{cases} a_F^1(S) & if \quad K = 1 \\ a_F^0(S) & if \quad K = 0 \end{cases}$$

while the associated EPQ is

$$P_F^K(S) = \begin{cases} [D_{FF}^1(S) - I_{FF}^1(S)] + [D_{FU}^1(S) - I_{FU}^1(S)] & if \quad K = 1 \\ 0 \quad if \quad K = 0 \end{cases}$$

Similarly, when demand is Unfavorable (i.e. in state U), the optimal production lot sizing policy and the associated total production-inventory costs during period 2 are

$$K = \begin{cases} 1 & if \quad a_U^1(S) < a_U^0(S) \\ 0 & if \quad a_U^1(S) \geq a_U^0(S) \end{cases}$$

$$C_2(U,S) = \begin{cases} a_U^1(S) & if \quad K = 1 \\ a_U^0(S) & if \quad K = 0 \end{cases}$$

In this case, the EPQ is

$$P_U^1(S) = \begin{cases} [D_{UF}^1(S) - I_{UF}^1(S)] + [D_{UU}^1(S) - I_{UU}^1(S)] & if \quad K = 1 \\ 0 \quad if \quad K = 0 \end{cases}$$

4. Implementation

4.1 Case Description

In order to demonstrate the use of the model in §2-3,a real case application from Mukwano Industries; a manufacturer of edible cooking oil in Uganda is presented in this section. Cooking oil is produced in three distinct sizes: Small: 5 litre Jerry cans(S=1), Medium: 10 litre Jerry cans(S=2), and Large: 20 litre Jerry cans(S=3).

Demand fluctuates every week and the factory wants to avoid excess inventory when demand is Unfavorable (state U), or running out of stock when demand is Favorable (state F).Decision support is sought in terms of an optimal production lot sizing policy, the associated production-inventory costs and specifically, a recommendation as to the EPQ of edible cooking oil for each respective size over the next two-week period is required.

4.2 Data Collection

Samples of customers were taken for the different sizes of cooking oil. Past data revealed the following demand pattern and inventory levels of the three sizes over twelve weeks.

Table 1
State-transitions, customers, demand and inventory levels of cooking oil sizes given production lot sizing policy 1 over 12 weeks

State transition (i,j)	Size (S)	Customers $N^K_{ij}(S)$	Demand $D^K_{ij}(S)$	Inventory $I^K_{ij}(S)$
FF	1	91	156	95
FU	1	71	115	93
UF	1	64	107	93
UU	1	13	11	94
FF	2	49	93	145
FU	2	55	60	145
UF	2	59	59	79
UU	2	13	11	80
FF	3	57	82	68
FU	3	62	93	69
UF	3	62	84	72
UU	3	9	9	69

Table 2
State-transitions, customers, demand and inventory levels of cooking oil sizes given production lot sizing policy 0 over 12 weeks

State transition (i,j)	Size (S)	Customers $N^K_{ij}(S)$	Demand $D^K_{ij}(S)$	Inventory $I^K_{ij}(S)$
FF	1	82	123	44
FU	1	50	78	45
UF	1	56	78	47
UU	1	25	15	46
FF	2	54	72	81
FU	2	46	77	79
UF	2	45	75	80
UU	2	11	11	79
FF	3	36	51	105
FU	3	53	70	100
UF	3	56	72	100
UU	3	9	10	50

The following unit production, holding and shortage costs(in UGX) were captured for each individual size of cooking oil at the production plant.

Small: 5 litre Jerry cans(S=1)
$c_p(1) = 4500$, $c_h(1) = 600$, $c_g(1) = 300$
Medium: 10 litre Jerry cans(S=2)
$c_p(2) = 4800$, $c_h(2) = 900$, $c_g(2) = 300$
Large: 20 litre Jerry cans(S=3)
$c_p(3) = 5100$, $c_h(3) = 1200$, $c_g(3) = 300$

4.3 Calculating $Q^K(S)$ and $C^K(S)$

Using (4), when additional units were produced (K=1),

$$Q^1(1) = \begin{bmatrix} 0.5617 & 0.4383 \\ 0.8312 & 0.1688 \end{bmatrix} \qquad Q^1(2) = \begin{bmatrix} 0.4660 & 0.5340 \\ 0.8194 & 0.1806 \end{bmatrix}$$

$$Q^1(3) = \begin{bmatrix} 0.4790 & 0.5210 \\ 0.8732 & 0.1268 \end{bmatrix}$$

When additional units were *not* produced (K=0),

$$Q^0(1) = \begin{bmatrix} 0.6212 & 0.3788 \\ 0.6914 & 0.3086 \end{bmatrix} \qquad Q^0(2) = \begin{bmatrix} 0.5400 & 0.4600 \\ 0.8036 & 0.1964 \end{bmatrix}$$

$$Q^0(3) = \begin{bmatrix} 0.4045 & 0.5955 \\ 0.8615 & 0.1385 \end{bmatrix}$$

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Using (5), when additional units were produced (K=1), the following production-inventory cost matrices are obtained:

$$C^1(1) = \begin{bmatrix} 0.3294 & 0.1189 \\ 0.0756 & 0.0498 \end{bmatrix} \quad C^1(2) = \begin{bmatrix} 0.0468 & 0.0765 \\ 0.0180 & 0.0621 \end{bmatrix}$$

$$C^1(3) = \begin{bmatrix} 0.0924 & 0.1584 \\ 0.0792 & 0.0720 \end{bmatrix}$$

When additional units were *not* produced (K=0), the following production-inventory cost matrices are obtained:

$$C^0(1) = \begin{bmatrix} 0.4266 & 0.1836 \\ 0.1674 & 0.0186 \end{bmatrix} \quad C^0(2) = \begin{bmatrix} 0.0081 & 0.0018 \\ 0.0045 & 0.0612 \end{bmatrix}$$

$$C^0(3) = \begin{bmatrix} 0.0648 & 0.0360 \\ 0.0336 & 0.0480 \end{bmatrix}$$

4.4 Calculating $e^K_i(S)$,$a^K_i(S,n)$ and $P^K(S)$

When additional units were produced (K=1), the following costs (in million UGX) result:

Small size (S=1)

$e^1_F(1) = (0.5617)(0.3294) + (0.4383)(0.1189)$
$= 0.2371$
$e^1_U(1) = (0.8312)(0.0756) + (0.1688)(0.0498)$
$= 0.0712$

Medium size(S=2)

$e^1_F(2) = (0.4660)(0.0468) + (0.5340)(0.0765)$
$= 0.0627$
$e^1_U(2) = (0.8194)(0.0180) + (0.1805)(0.0621)$
$= 0.0260$

Large size(S=3)

$e^1_F(3) = (0.4790)(0.0924) + (0.5210)(0.1584)$
$= 0.1268$
$e^1_U(3) = (0.8732)(0.0792) + (0.1268)(0.0720)$
$= 0.0783$

However, when additional units were *not* produced (K=0), the following costs result(in million UGX)

Small size (S=1)

$e^0_F(1) = (0.6212)(0.4266) + (0.3788)(0.1836)$
$= 0.3346$
$e^0_U(1) = (0.6914)(0.1674) + (0.3086)(0.0186)$
$= 0.1215$

Medium size(S=2)

$e^0_F(2) = (0.5400)(0.0081) + (0.4600)(0.0018)$
$= 0.0052$
$e^0_U(2) = (0.8036)(0.0045) + (0.1964)(0.0621)$
$= 0.0156$

Large size(S=3)

$e^0_F(3) = (0.4095)(0.0648) + (0.5955)(0.0360)$
$= 0.0480$
$e^0_U(3) = (0.8615)(0.0336) + (0.1385)(0.0480)$
$= 0.0360$

The cumulative production-inventory costs and economic production quantity are computed using (1) and (6). Results are summarized in Table 3.

Table 3

Values of K,$a^K_i(S,n)$ and $P^K_i(S)$ for different sizes of cooking oil during week 2

Size of Jerry can (S)	Production Lot sizing Policy (K)	
Small (1)	K=1	K=0
$a^K_F(1,2)$	0.4015	0.5086
$a^K_U(1,2)$	0.2803	0.3074
$P^K_F(1)$	83	0
$P^K_U(1)$	14	0
Medium (2)	K=1	K=0
$a^K_F(2,2)$	0.2112	0.1660
$a^K_U(2,2)$	0.2331	0.2197
$P^K_F(2)$	0	0
$P^K_U(2)$	0	0
Large (3)	K=1	K=0
$a^K_F(3,2)$	0.2775	0.1863
$a^K_U(3,2)$	0.2943	0.2501
$P^K_F(3)$	0	0
$P^K_U(3)$	0	0

4.5 The Optimal Production lot sizing policy and EPQ

Week 1

Small size -5 Litre Jerry cans

Since 0.2371 < 0.3346, it follows that K=1 is an optimal production lot sizing policy for week 1 with associated total production-inventory costs of 0.2371 million UGX and an EPQ of (156-95)+(115-93) = 83 units when demand is favorable.

Since 0.0712 < 0.1215, it follows that K=1 is an optimal production lot sizing policy for week 1 with associated total production-inventory costs of 0.0712 million UGX and an EPQ of (107-93) = 14 units when demand is unfavorable.

Medium size -10 Litre Jerry cans

Since 0.0052 < 0.0627, it follows that K=0 is an optimal production lot sizing policy for week 1 with associated total production-inventory costs of 0.0052 million when demand is favorable.

Since 0.0156<0.0260, it follows that K=0 is an optimal production lot sizing policy for week 1 with associated total production-inventory costs of 0.0156 million UGX when demand is unfavorable.

EPQ=0 regardless of the state of demand.

Large size -20 Litre Jerry cans

Since 0.0480 < 0.1268, it follows that K=0 is an optimal production lot sizing policy for week 1 with associated total production-inventory costs of 0.0480 million when demand is favorable.

Since 0.0360 < 0.0783, it follows that K=0 is an optimal production lot sizing policy for week 1 with associated total production-inventory costs of 0.0360 million UGX when demand is unfavorable.

EPQ=0 regardless of the state of demand.

Week 2

Small size -5 Litre Jerry cans

Since 0.4015 < 0.5086, it follows that K=1 is an optimal production lot sizing policy for week 2 with associated accumulated production-inventory costs of 0.4015 million UGX and an EPQ of (156-95) + (115-93) = 83 units when demand is favorable.

Since 0.2803 < 0.3074, it follows that K=1 is an optimal production lot sizing policy for week 2 with associated accumulated production-inventory costs of 0.2803 million UGX and an EPQ of (107-93) = 14 units when demand is unfavorable.

Medium size -10 Litre Jerry cans

Since 0.1660 < 0.2112, it follows that K=0 is an optimal production lot sizing policy for week 2 with associated accumulated production-inventory costs of 0.1660 million when demand is favorable.

Since 0.2197 < 0.2331, it follows that K=0 is an optimal production lot sizing policy for week 2 with associated total production-inventory costs of 0.2197 million UGX when demand is unfavorable.

EPQ=0 regardless of the state of demand.

Large size -20 Litre Jerry cans

Since 0.1863 < 0.2775, it follows that K=0 is an optimal production lot sizing policy for week 2 with associated accumulated production-inventory costs of 0.1863 million when demand is favorable.

Since 0.2501 < 0.2943, it follows that K=0 is an optimal production lot sizing policy for week 2 with associated accumulated production-inventory costs of 0.2501 million UGX when demand is unfavorable.

EPQ=0 regardless of the state of demand.

5. Conclusion

An optimization model was presented in this paper. The model determines an optimal production lot sizing policy, production-inventory costs and the EPQ of an item with varying size and stochastic demand. The decision of whether or not to produce additional units of a specific size of item is modeled as a multi-period decision problem using dynamic programming over a finite period planning horizon. The working of the model was demonstrated by means of a real case study. It is equally important to examine the behavior of EPQ for the respective sizes of cooking oil under non stationary demand conditions. In the same spirit, the model raises a number of salient issues to consider: Production disruptions in a typical manufacturing set up and customer response to abrupt changes in price of cooking oil. Finally, special interest is sought in extending the model to optimize EPQ using Continuous Time Markov Chains (CTMC) in conjunction with dynamic programming.

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