

Analysis of impact of firing angle on an AC chopper in terms of Harmonic Distortion & Power factor

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Abstract: The phase angle controlled AC choppers are widely used for voltage regulation & controlling. Apart from the advantages of method such as low cost I Simple construction, easy operation it has a major disadvantage of inducing Harmonic components & power factor even with pure resistive load. Hence in this paper we are analyzing the various aspects of AC choppers by Simulation & mathematical analysis. The AC chopper considered in this paper is symmetric & the Switching devices are considered to have ideal Switching characteristics for mathematical Analysis. The Simulation & graphical analysis are performed in MALAB/ Simulink.

Keywords: AC choppers, Harmonic Distortion, Power Factor.

1. Introduction

AC choppers are widely used as AC regulator or to obtain variable voltage from a fixed power Supply as a cheaper, Smaller & Controllable replacement of transformer based systems. It is largely used in industrial heating, lighting controls and motor speed controllers because of above discussed advantages. However the controlling technique which basically works on abruptly Switching of Controlling devices Such as Triac, SCR etc. induces discontinuities & non-linearity which ultimately causes Significant harmonic in load current, a lagging power factor & some transient phenomenon can also be observed. The effect of chipping a waveform gets move & move complicated as the number of phases increases however in this paper we restrict the Study to Single phase symmetrical AC chopper as Shown in figure 1.

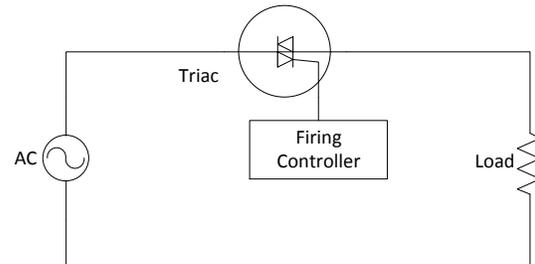


Figure 1: Basic PAC, AC chopper topology used in this paper.

Since this paper involves harmonic analysis hence in depth discussion on Fourier Series is performed in next Section.

2. Fourier Series

The fundamentals of Fourier series and of the harmonic concept are presented in detail in Ref. (66). Some elements of this concept that are relevant to electrical circuits and the symbols used are explained below.

Electrical quantities such as voltages $u(t)$, currents $i(t)$ or fluxes $\varphi(t)$ are periodic if for any instant of time t they satisfy the relation

$$u(t) = u(t \pm nT) \dots \dots (1)$$

Where n is any integer and T , called a period, is a nonzero real number. An example of a periodic quantity, $x(t)$, is shown in Figure. 2. Mathematically, the period T is the smallest number that satisfies Eq. (1). This condition is often neglected in electrical engineering. In particular, the period T of a power system voltage is usually considered to be the period of other periodic quantities in such a system, even if Eq.

(1) is satisfied also for a shorter time. For example, the output voltage of a six pulse ac-dc converter satisfies Eq. (1) also for $T/6$, but it is usually considered as a periodic quantity with the period T not $T/6$.

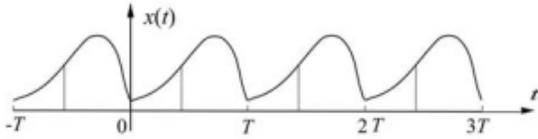


Figure 2: Periodic quantity $x(t)$ of the period T .

Periodic quantities in electrical systems are of a finite power, this means they are integrable with square, i.e.,

$$\frac{1}{T} \int_0^T f^2(t) dt < \infty \dots\dots\dots (2)$$

Hence the function is representable by Fourier Series, since the Series will converge. Such a function can be represented by Fourier series as follows

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t \dots (3)$$

Where the a_0 , a_n & b_n are the Fourier coefficients & can be estimated as

$$a_0 = \frac{1}{T} \int_0^T f(t) dt \dots\dots\dots (4)$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos(n\omega t) dt \dots\dots\dots (5)$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin(n\omega t) dt \dots\dots\dots (6)$$

The above stated equations represents that each frequency component is made of two orthogonal Components of Same frequency which are in form of Sine & cosine waves Hence the Complete amplitude & phase angle of any frequency component can be given as

$$C_n = \sqrt{a_n^2 + b_n^2} \dots\dots\dots (7)$$

$$\phi = \tan^{-1} \left(\frac{b_n}{a_n} \right) \dots\dots\dots (8)$$

And finally the equation 7 & 8 could be represented in more understandable format by $C_n \sin(n\omega t + \phi) \dots\dots\dots (9)$

The equation 9 represents the n th harmonics with its amplitude & phase.

Phase Angle Controlled (PAC) AC chopper.

As in early sections that the PAC, AC chopper works by chopping the wave at certain angle the figure below Shows the firing pulses & corresponding chopped Waveform across the load for voltage & current

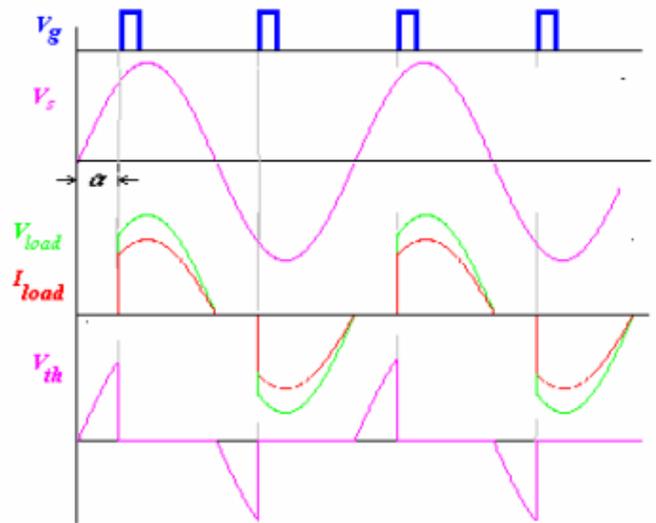


Figure 3: Waveforms to AC chopper on resistive load.

From the waveforms the V_{rms} (Root mean Squared voltage) which decides the power Supplied to the load can be calculated as

$$V_{rms} = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\pi} 2V^2 \sin^2(\omega t) d\omega t} \dots\dots\dots 10$$

$$V_{rms} = V \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}} \dots\dots\dots 11$$

In the above expression the V represents the rms value of the input voltage & V_{rms} represents the rms value of output voltage. Here it must be understood that it may not be the full utilizable power because it is the SUM of all harmonic components & fundamental useful component (in case of electrical machines like motors not for resistive heating elements) hence the useful component will always be less than it. The value of fundamental component which is evaluated by the help of Fourier series as explained in section 2 is given below

$$V_{fund} = \frac{\sqrt{2}V}{\pi} \left[\left(\pi - \frac{\alpha}{2} + \frac{\sin 2\alpha}{2} \right) \sin \omega t - \left(\frac{1}{2} + \frac{\cos 2\alpha}{2} \right) \cos \omega t \right] \dots\dots\dots 12$$

So the equation 12 clearly explains the effect of chopping angle on fundamental Component & the difference of equation 11 & 12 can be used to estimate the produced harmonics components.

Another important aspect of AC systems is power factor which is quite Simple for linear System & can be represented by the cosine of the angle difference between voltage & current for example let the voltage & current equation for any linear System are 13 & 14 respectively

$$v(t) = V_{peak} \sin(\omega + \delta) \dots\dots\dots 13$$

$$i(t) = I_{peak} \sin(\omega t + \theta) \dots\dots\dots 14$$

Then the power factor of the system can be defined as

$$pf = \cos(\delta - \theta) \dots\dots\dots 14$$

But for the system which contains non-linearly the Situation will be move complex because it Contains large number of Harmonic components hence another move generalized definition of power factor is required & it can be stated as the ratio of average power to appetent power hence

Power factor = average power /apparent power.....15

For nonlinear systems voltage & current can be represented by Sum of harmonic components evaluated by fairies Series as given below

$$v(t) = \sum_{k=1}^{\infty} V_k \sin(k\omega t + \delta_k) \dots\dots\dots 16$$

$$i(t) = \sum_{R=1}^{\infty} I_k \sin(k\omega t + \theta_k) \dots\dots\dots 17$$

For calculation we need apparent power which is the product of rms value of voltage & current. For the voltage & current shown in equation 16 & 17 could be represented by the Sum of rms values of each harmonic Component at

$$V_{rms} = \sqrt{\sum_{k=1}^{\infty} \frac{V_k^2}{2}} \dots\dots\dots 18$$

$$i_{rms} = \sqrt{\sum_{k=1}^{\infty} \frac{I_k^2}{2}} \dots\dots\dots 19$$

Same could be used for average power calculation

$$P_{avg} = \sum_{k=1}^{\infty} v_{rms} i_{rms} \cos(\delta_k - \theta_k) \dots\dots\dots 20$$

Hence the true power factor can be calculated as

$$pf = \frac{P_{avg}}{V_{rms} i_{rms}} \dots\dots\dots 21$$

One move common way of presenting is in terms of Total Harmonic Distortion (THD) using THD we can calculate the rms value of chopped wave in terms of rms value of fundamental component

$$V_{rms} = v_{1rms} \sqrt{1 + (THD_v)^2} \dots\dots\dots 22$$

$$i_{rms} = i_{1rms} \sqrt{1 + (THD_i)^2} \dots\dots\dots 23$$

Now substituting equation 22 & 23 into equation 21

$$pf = \frac{P_{avg}}{v_{1rms} i_{1rms} \sqrt{1 + (THD_v)^2} \sqrt{1 + (THD_i)^2}} \dots\dots\dots 24$$

The equation in can be Simplified by taking assumption that average power is approximately equal to average power of fundamental Component & the distortion in voltage is very small ten the equation au can be rewritten as

$$pf = \frac{P_{1avg}}{v_{1rms} i_{1rms} \sqrt{1 + (THD_i)^2}} \dots\dots\dots 25$$

A deeper inspection of above equation shows that its first term is equal to displacement power factor (power factor of fundamental component) & the Second & last term represents the distortion power factor which is given by

$$pf_{dist} = \frac{i_{1rms}}{i_{rms}} \dots\dots\dots 26$$

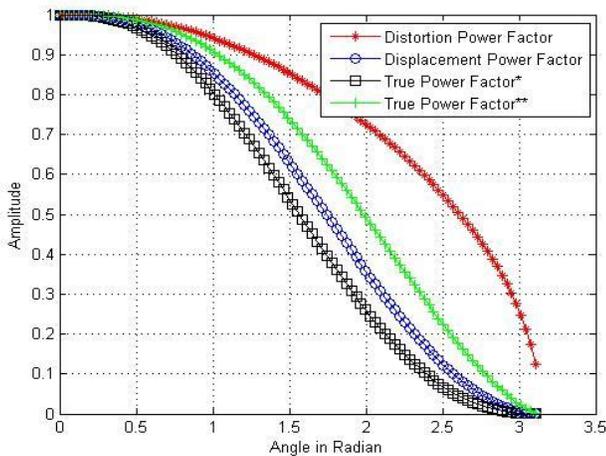
hence it can be again modify to

$$pf_{true} = pf_{disp} \times pf_{dist} \dots\dots\dots 27$$

Since distortion power factor cannot be greater than one hence we can say that true power will always be less than the displacement power for nonlinear loads.

Simulation Results

The discussion in previous Section results some important relations which are useful for estimating the effect of chopping angle in various quantities such as true power factor, total harmonic distortion & distortion factor etc. in this Section we simulated all these equations & results are plotted for making some useful conclusion.



*Harmonic Components are considered reactive (useless) [1].
**Harmonic Components are not considered reactive (useless) [1].

Figure 4: Plot of Different Power Factor for varying triggering angle α .

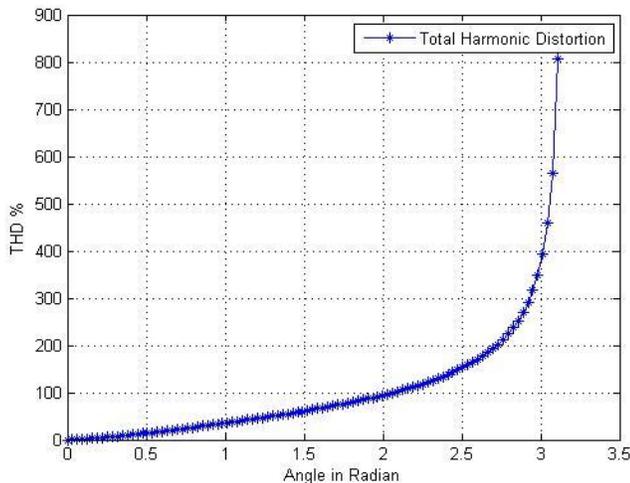


Figure 5: Variation of THD in Percentage with triggering angle.

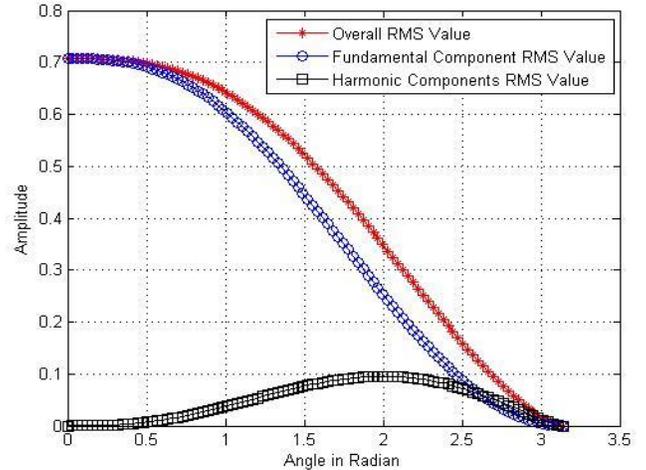


Figure 6: Comparison of RMS values for Chopped wave & Fundamental Component with variation of triggering angle.

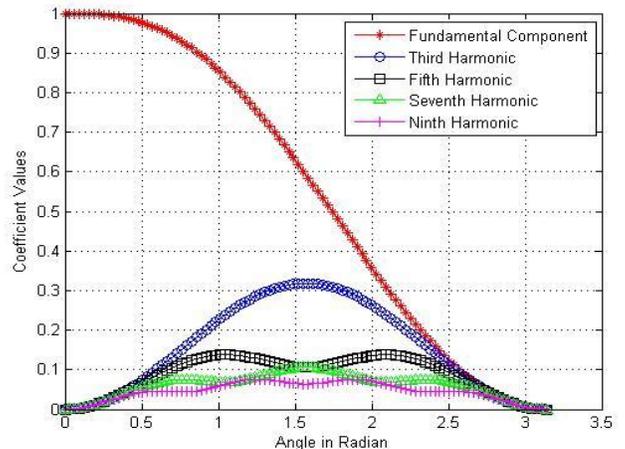


Figure 7: Comparison of RMS values for Fundamental Component and various Harmonic components with variation of triggering angle.

Conclusion

The plot shown in figure 4 true power factor* and true power factor** are two different considerations first in which except fundamental components all are considered reactive here reactive doesn't mean real reactive instead the non-useful components which is not always true (like in case of resistive heating etc.) hence the second plot is generated in which other components are considered as their own power factors. Finally the figure 4 helps to conclude that the true power factor* will always be less than the displacement power factor in case of chopped wave.

Figure 5 shows the variation in THD value which gradually rises at starting values of firing angles but rises quickly as the angle reaches to 180 degree this not because of increase in harmonics but reduction in the fundamental component as the figure 6 clarify that at higher angles fundamental component decreases more sharply than overall RMS value.

The figure 7 represents more rigorous analysis in terms of variations in different harmonic components with respect to firing angles. As described in section 2 the function of chopped wave considered in this particular study is odd hence the even components are missing and only odd components are plotted. From the plot rise of harmonic components can be seen around 90 degree of the firing angle hence at these angles special attention is needed this increase in harmonic components can also be seen in figure 6.

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