The effect of rotation and magnetic field on thermal instability of compressible Walters' B′ viscoelastic fluid

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ABSTRACT

The thermal convection of compressible Walters'B' Rotating viscoelastic fluid is considered in the presence of uniform vertical magnetic field. By applying normal mode analysis method, the dispersion relation has been derived and solved analytically. For stationary convection, Walters'B' viscoelastic fluid behaves like an ordinary (Newtonian) fluid. It is found that rotation has a stabilizing effect, whereas the magnetic field has both stabilizing and destabilizing effects. The rotation and magnetic field are found to introduce oscillatory mode in the system which were non – existent in their absence. The results are presented through graphs.

Key words: Walters'B′ Fluid, thermal convection, viscoelasticity, Rotation, Magnetic field.

INTRODUCTION

A detailed account of the theoretical and experimental study of thermal instability in Newtonian fluids under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar[3]. The thermal instability of a fluid layer with maintained adverse temperature gradient by heating the underside plays an important role in geophysics, interior of the Earth, Oceanography and the atmospheric physics have investigated by Rayleigh [11] and Jeffreys [5]. Chandra [4] has employed instability of fluid heated from below. Bhatia and Steiner[2] have studied the problem of thermal instability of a maxwellian viscoelastic fluid in the presence of the rotation and have found that a rotation has a destabilizing effect , in contrast to the stabilizing effect on Newtonian viscous fluid. Sharma [13] has studied the thermal instability of a layer of oldroydian viscoelastic fluid acted on by a uniform rotation and found that rotation has destabilizing as well as stabilizing effect under certain conditions in contrast to that of a maxwell fluid where it has a destabilizing effect. In

another study, Sharma [12] has studied thermal instability in a viscoelastic fluid in the hydromagnetics. Oldroyd [9] investigated non – Newtonian effects in study motion of some idealized elastic – viscous liquids. Kent [6] studied instability of laminar flow of a magnetofluid. Veronis[17] analysed connective instability in a compressible atmosphere.

Sharma[14] investigated the thermal instability of compressible fluids in the presence of rotation and magnetic field. Kumar et al. [7] have studied the thermal convection in a walters'(model B′) elastico – viscous dusty fluid in hydromagnetics with the effects of compressibility and rotation. In a recent study, Rana and Kango[10] have considered the thermal instability of compressible walters'(model B′) rotating fluid in the presence of suspended particles and the porous medium. Spiegel and Veronis[16] have studied on the boussinesq approximation for compressible fluid. Kumar et al. [8] have investigated the thermal instability of walters' B′ visoelastic fluid permeated with suspended particles in hydromagnetics in porous medium. Walters [18] has studied the motion of an elastic – viscous liquid contained between concentric spheres. In another study, Walters [19] has studied non – Newtonian effects in some elastico – viscous liquids whose behavior at small rates of shear is characterized by a general linear equation of a state. Sharma et al. [15] has analyzed the thermal instability of walters'(model B′) elastico – viscous fluid in the presence of variable gravity field and rotation in porous medium. Recently, Aggarwal and Anushri verma [1] have considered the effect of rotation and the magnetic field on thermal instability of viscoelastic fluid permeated with suspended particles.

There is growing importance of non – Newtonian fluids, convection in fluid layer heated from below, magnetic field, and rotation, the present paper attempts to study the effect of uniform vertical magnetic field on Non - Newtonian

viscoelastic fluid heated from below in the presence of a uniform rotation.

MATHEMATICAL MODEL AND PERTURBATION EQUATIONS

Consider an infinite, horizontal, compressible electrically conducting Walters'B′ viscoelastic fluid layer of thickness d, heated from below so that the temperatures and densities at the bottom surface z = 0 are T_0 and ρ_0 and at the upper surface z = d are T_d and ρ_d, respectively, and that a uniform temperature gradient β(= |dT/dz|) is maintained. The gravity field **g**(0,0,-g), a uniform vertical magnetic field **H**(0,0,H), and a uniform vertical rotation **Ω**(0,0,Ω) act on the system.

The equations of motion, continuity, heat conduction, and Maxwell's equations governing the flow of Walters'B′ viscoelastic fluid in the presence of magnetic field and rotation are

$$\frac{\partial q}{\partial t} + (q.\nabla)q = -\nabla(\frac{p}{\rho_0} - \frac{1}{2}|\Omega \times r|^2) + g\left(1 + \frac{\delta\rho}{\rho_0}\right) + \left(v - v'\frac{\partial}{\partial t}\right)\nabla^2 q + \frac{\mu_e}{4\pi\rho_0}(\nabla \times H) \times H + 2(q \times \Omega),$$
(1)

$$\frac{\partial q}{\partial t} + \nabla.(\rho q) = 0,$$
(2)

$$\frac{\partial T}{\partial t} + (q \times \nabla)T = \chi\nabla^2 T,$$
(3)

$$\nabla.H = 0,$$
(4)

$$\frac{\partial H}{\partial t} = (H.\nabla)q + \eta\nabla^2 H$$
(5)

where **q**(u,v,w), p, ρ, T, υ, and υ' denote the velocity, pressure, density, temperature, kinematic viscosity, and kinematic viscoelasticity, respectively, and **r**(x,y,z).

The equation of the fluid state is

$$\rho = \rho_0[1 - \alpha(T-T_0)],$$
(6)

where ρ_0, T_0 are, respectively, the density and temperature of the fluid at the reference level z = 0 and α is the coefficient of thermal expansion. From equation (1), we made use of the Boussinesq approximation, which states that the density variations are ignored in all terms in the equations of motion except the external force term. The magnetic permeability μ_e, thermal diffusivity χ, and electrical resistivity η are all assumed to be constant.

The basic state is one in which the velocity, density, pressure, and temperature at any point in the

fluid are, respectively, given by

$$\mathbf{q}= (0,0,0), \qquad \rho=\rho(z), \ \ p=p(z), \ \ T=T(z) \qquad (7)$$

Let **q**(u,v,w), δp, δρ, θ, and **h**(h_x,h_y,h_z) denote, respectively, the perturbations in velocity **q** (initially zero), pressure p, density ρ, temperature T, and the magnetic field **H**(0,0,H). The change in density δρ, caused by the perturbation θ in temperature, is given by

$$\delta_\rho = -\alpha\rho_0\theta \qquad (8)$$

Following the assumptions given by Spiegel and Veronis[14] and the results for compressible fluid, the flow equations are found to be the same as that of incompressible fluid except that the static temperature gradient β is replaced by the excess over adiabatic $(\beta - \frac{g}{c_p})$, c_p being specific heat of the fluid at constant pressure.

Then the linearized perturbation equations

$$\frac{\partial q}{\partial t} = \frac{-1}{\rho_0}(\nabla\delta p) - g\alpha\Theta + \left(v - v'\frac{\partial}{\partial t}\right)\nabla^2 q + \frac{\mu_e}{4\pi\rho_0}(\nabla \times h) \times H + 2(q \times \Omega)$$

$$\nabla.q = 0$$

$$\frac{\partial\theta}{\partial t} = (\beta - \frac{g}{c_p})w + \chi\nabla^2\theta \qquad (9)$$

$$\nabla.h = 0,$$

$$\frac{\partial h}{\partial t} = (H.\nabla)q + \eta\nabla^2 h$$

Within the framework of Boussinesq approximation, (9) become

$$\frac{\partial}{\partial t}\nabla^2 = \left(v - v'\frac{\partial}{\partial t}\right)\nabla^4 w + \frac{\mu_e}{4\pi\rho_0}\nabla^2\left(\frac{\partial h_z}{\partial z}\right) + g\alpha\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) - 2\Omega\frac{\partial\zeta}{\partial z},$$

$$\frac{\partial\zeta}{\partial z} = \left(v - v'\frac{\partial}{\partial t}\right)\nabla^2\zeta + 2\Omega\frac{\partial w}{\partial z} - \frac{\mu_e H}{4\pi\rho_0}\frac{\partial\xi}{\partial z}$$

$$\left(\frac{\partial}{\partial t} - \chi\nabla^2\right)\theta = (\beta - \frac{g}{c_p})w \qquad (10)$$

$$\left(\frac{\partial}{\partial t} - \eta\nabla^2\right)h_z = H\frac{\partial w}{\partial z},$$

$$\left[\frac{\partial}{\partial t} - \eta\nabla^2\right]\xi = H\frac{g\zeta}{\partial x},$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ and $\zeta = \partial v/\partial x - \partial u/\partial y$; $\xi = \partial h_y/\partial x - \partial h_x/\partial y$ stand for the z-components of vorticity and current density, respectively.

THE DISPERSION RELATION

Analyzing the disturbances into normal modes, We assume that that the perturbation quantities are of the form

[w, θ, h$_z$, ζ,ξ] = [w(z), Θ(z), K(z), Z(z), X(z)] exp (ik$_x$ x + ik$_y$y+nt),   (11)

where k_x, k_y *are the wave numbers along x and y directions,* respectively, $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number, and *n is the growth rate which is, in general, a complex constant.*

Using equation (11), (12) in non-dimensional form transform to

$$[\sigma(D^2 - a^2)^2\, W + \left(\frac{g\alpha d^2}{v}\right)a^2\Theta + \frac{2\Omega d^3}{v}\,DZ - \frac{\mu_e Hd}{4\pi\rho_0 v}\,(D^2 - a^2)DK = [1 - F\sigma](D^2 - a^2)^2 W,$$
(12)

$$[(1 - F\sigma)\,(D^2 - a^2) - \sigma]Z = -\left(\frac{2\Omega d}{v}\right)DW - \frac{\mu_e Hd}{4\pi\rho_0}\,DX],$$
(13)

$$[D^2 - a^2 - p_1\sigma]\Theta = -\frac{(\beta - \frac{g}{c_p})d^2}{\chi}W,$$
(14)

$$[D^2 - a^2 - p_2\sigma]k = -\left(\frac{Hd}{\eta}\right)DW,$$
(15)

$$[D^2 - a^2 - p_2\sigma]X = -\left(\frac{Hd}{\eta}\right)DZ,$$
(16)

where we have introduced new coordinates *(x' ,y' ,z') = (x/d,y/d,z/d)* in new units of length *d* and   *D = d/dz'.* For convenience, the dashes are dropped hereafter. Also we have put $a = kd$, $\sigma = nd^2/v$, $F = v'/d^2$; $p_1 = v/\chi$ is the Prandtl number and $p_2 = v/\eta$ is the magnetic Prandtl number.

Consider the case where both the boundaries are free as well as perfect conductors of heat, while the adjoining medium is also perfectly conducting. The case of two free boundaries is slightly artificial, except in stellar atmospheres and in certain geophysical situations where it is most appropriate. However, the case of two free boundaries allows us to obtain analytical solution without affecting the essential features of the problem. The appropriate boundary conditions, with respect to which (12) - (16) must be solved, are

W = D^{2}W = 0,   DZ = 0,   Θ= 0   at z = 0, z=l, (17)

DX = 0, K = 0, on a perfectly conducting boundary.

Using the above boundary conditions, it can be shown that all the even-order derivatives of *W* must vanish for *z = 0* and *z = 1*, and hence the proper solution of *W* charac-terizing the lowest mode is

W = W$_0$ sin πz,   where W$_0$ is a constant.   (18)

Eliminating Θ, *K, Z,* and *X* between(12)–(16) and substituting(18) in the resultant equation, we obtain the dispersion relation

$$R_1\left(\frac{G}{G-1}\right)\left(\frac{1+x}{x}\right)\frac{[[(1 - iF_1\sigma_1\pi^2)(1+x)+i\sigma_1\}\{1+x+i\sigma_1 p_2\}+Q_1][1+x+i\sigma_1 p_1]}{(1+x+i\sigma_1 p_2)}$$

$$+ \frac{T_1(1+x+i\sigma_1 p_2)(1+x+i\sigma_1 p_1)}{\{x[((1 - iF_1\sigma_1\pi^2)(1+x)+i\sigma_1\}\{1+x+i\sigma_1 p_2\}+Q_1]\}}\,].$$
(19)

Where R = gαβd^4/νχ, Q = μ$_e$H^{2}d^2/4πρ$_0$νη, T$_A$ = 4Ω^{2}d^4/ν^2 stand for the Rayleigh-number, the Chandrasekhar number, the Taylor number, respectively. Putting

$$x = \frac{a^2}{\pi^2},\quad R_1 = \frac{R}{\pi^4},\quad i\sigma_1 = \frac{\sigma}{\pi^2}, F_1 = \pi^2 F,$$
$$T_1 = \frac{T_A}{\pi^4},\qquad Q_1 = \frac{Q}{\pi^2},\qquad i = \sqrt{-1},$$
(20)

Where $G = (c_p\frac{\beta}{g})$ is the dimensionless compressibility.

THE STATIONARY CONVECTION

When the instability sets in as stationary convection, the marginal state will be characterized by σ = 0. Putting σ = 0, the dispersion relation (19) reduces to

$$R_1 = \left(\frac{G}{G-1}\right)\left[\left(\frac{1+x}{x}\right)\left[(1+x)^2 + Q_1\right] + \frac{T_1(1+x)^2}{x[(1+x)^2+Q_1]}\right]$$
(21)

We thus find that for the stationary convection, the viscoelasticity parameter F vanishes with σ and Walters'B′ viscoelastic fluid behaves like an ordinary Newtonian fluid.

To study the effects of rotation and magnetic field, we examine the natures of $\frac{dR_1}{dQ_1}$ analytically.

Equation (21) yields

$$\frac{dR_1}{dT_1} = \left(\frac{G}{G-1}\right)\left[\frac{(1+x)^2}{x[(1+x)^2+Q_1]}\right]$$
(22)

$$\frac{dR_1}{dQ_1} = \left(\frac{G}{G-1}\right)\left[\frac{(1+x)}{x} - \frac{T_1(1+x)^2}{x[(1+x)^2+Q_1]^2}\right]$$
(23)

It is evident from (22) that for a stationary convection, $\frac{dR_1}{dT_1}$ is always positive, thus, the rotation has a stabilizing effect on the system.
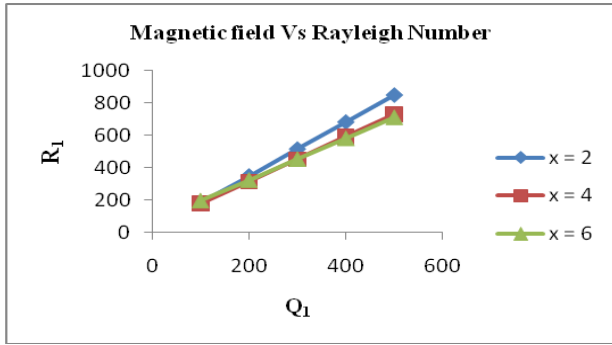
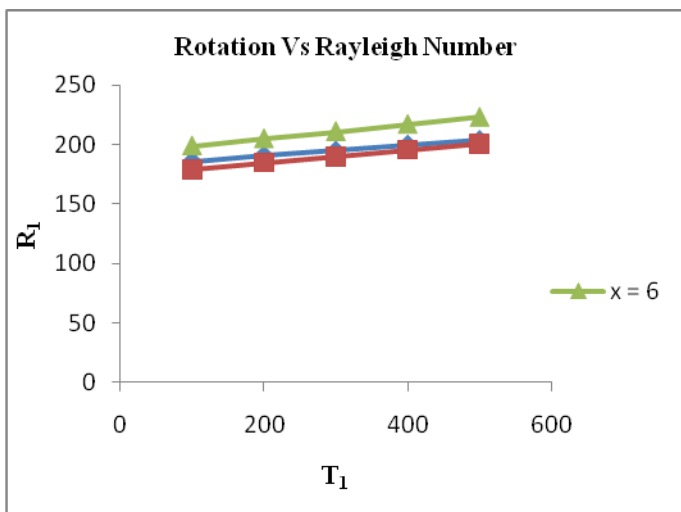**Figure 1** The Variation of R_1 With Q_1 for fixed values of $T_1 =$ 100, G = 10 and x = 2,4, 6.

Figure 2 The Variation of R_1 With T_1 for fixed values of $Q_1 =$ 100, G = 10 and x = 2,4, 6

It is clear from (23) for a stationary convection, dR_1/dQ_1 may be positive as well as negative, thus, the magnetic field has both stabilizing and destabilizing effects on the system.

The dispersion relation (21) is also analyzed numerically. Figure (1) shows the variation of R_1 with respect to Q_1, for fixed values of $T_1 = 100$, G =10 and wave numbers $x = 2,4, 6$ In Figure (2), R_1 is plotted against T_1, for fixed values of $Q_1 = 100$, G = 10and wave numbers $x = 2,4, 6$. The Rayleigh number R_1 increases with increase in rotation parameter T_1 showing its stabilizing effect on the system. It clearly depicts both the stabilizing and destabilizing effects of the magnetic field on the system.

STABILITY OF THE SYSTEM AND OSCILLATORY MODES

Multiplying (12) by $W*$, the complex conjugate of W, integrating over the range of z and using
(13)–(16), together with the boundary conditions (17), We obtain

$$-\sigma I_1 + \frac{g\alpha\chi a^2}{v\beta}\left(\frac{G}{G-1}\right)(I_2 + p_1\sigma^* I_3) - d^2(1 - F\sigma^*)I_4 -$$
$$d^2\sigma^* I_5 - \frac{\mu_e d^2\eta}{4\pi\rho_0 v}(I_6 + p_2\sigma I_7) - \frac{\mu_e\eta}{4\pi\rho_0 v}(I_8 + p_2\sigma^* I_9) =$$
$$(I - F\sigma)I_{10},$$

(24)

Where,

$$I_1 = \int_0^1 (|DW|^2 + a^2|W|^2)dz, \qquad I_2 = \int_0^1(|D\Theta|^2 + a^2|\Theta|^2)dz,$$

$$I_3 = \int_0^1(|\Theta|^2)dz, \quad I_4 = \int_0^1(|DZ|^2 + a^2|Z|^2)dz,$$

$$I_5 = \int_0^1(|Z|^2)dz, \qquad I_6 = \int_0^1(|DX|^2 + a^2|X|^2)dz,$$

$$I_7 = \int_0^1(|X|^2)dz, \quad I_8 = \int_0^1(|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2)dz$$

$$I_9 = \int_0^1(|DK|^2 + a^2|K|^2)dz, I_{10} = \int_0^1(|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2)dz,$$

(25)

and $\sigma*$ is the complex conjugate of σ. The integrals I_1.......I_{10} are all positive definite. Putting

$\sigma = \sigma_r + i\sigma_i$, where σ_r, σ_i are real. Separating the real and imaginary parts of (24), we obtain

$$\sigma_r\left[-I_1 + \frac{g\alpha\chi a^2}{v\beta}\left(\frac{G}{G-1}\right)I_3 p_1 - d^2 F I_4 - d^2 I_5 - \frac{\mu_e d^2\eta}{4\pi\rho_0 v}p_2 I_7 - \frac{\mu_e\eta}{4\pi\rho_0 v}p_2 I_9 - F I_{10}\right]$$

(26)

$$= -\frac{g\alpha\chi a^2}{v\beta}I_2\left(\frac{G}{G-1}\right) + d^2 I_4 + \frac{\mu_e d^2\eta}{4\pi\rho_0 v}I_6 + \frac{\mu_e\eta}{4\pi\rho_0 v}I_8 + I_{10},$$

$$\sigma_i\left[I_1 + \frac{g\alpha\chi a^2}{v\beta}\left(\frac{G}{G-1}\right)p_1 I_3 - d^2 F I_4 - d^2 I_5 - \frac{\mu_e d^2\eta}{4\pi\rho_0 v}p_2 I_7 - \frac{\mu_e\eta}{4\pi\rho_0 v}p_2 I_9 + F I_{10}\right] = 0$$

(27)

From (27), it is clear that σ_i is zero when the quantity multiplying it is not zero and arbitrary when this quantity is zero. If $\sigma_i \neq 0$, then (27) gives

$$\frac{ga\chi a^2}{v\beta}\left(\frac{G}{G-1}\right)p_1 I_3 - d^2 F I_4 - d^2 I_5 - \frac{\mu_e \eta}{4\pi\rho_0 v}p_2 I_9 = -I_1 - \frac{\mu_e d^2 \eta}{4\pi\rho_0 v}p_2 I_7 - F I_{10}$$

(28)

Substituting in (26), we have

$$I_{10} + \frac{\mu_e \eta}{4\pi\rho_0 v}I_8 + \frac{\mu_e \eta d^2}{4\pi\rho_0 v}I_6 + d^2 I_4 + 2\sigma_r\left[I_1 + \frac{\mu_e d^2 \eta}{4\pi\rho_0 v}p_2 I_7 + F I_{10}\right] = \frac{ga\chi a^2}{v\beta}\left(\frac{G}{G-1}\right)I_2$$

(29)

On applying Rayleigh-Ritzinequality, Equation (29) gives

$$\frac{(\pi^2+a^2)^3}{a^2}\int_0^1 |w|^2\, dz + \frac{(\pi^2+a^2)}{a^2} \times \left\{\frac{\mu_e \eta}{4\pi\rho_0 v}I_8 + \frac{\mu_e d^2 \eta}{4\pi\rho_0 v}I_6 + d^2 I_4 + 2\sigma_r\left[I_1 + \frac{\mu_e d^2 \eta}{4\pi\rho_0 v}p_2 I_7 + F I_{10}\right]\right\} \leq \frac{ga\chi}{v\beta}\int_0^1 |w|^2\, dz$$

(30)

Hence equation (30) yields

$$\left[\frac{27\pi^4}{4} - \frac{ga\chi}{v\beta}\right]\int_0^1 |w|^2\, dz + \frac{(\pi^2+a^2)}{a^2}$$

$$\times \left\{\frac{\mu_e \eta}{4\pi\rho_0 v}I_8 + \frac{\mu_e d^2 \eta}{4\pi\rho_0 v}I_6 + d^2 I_4 + 2\sigma_r\left[I_1 + \frac{\mu_e d^2 \eta}{4\pi\rho_0 v}p_2 I_7 + F I_{10}\right]\right\} \leq 0$$

(31)

since minimum value of $(\pi^2 + a^2)^3/a^2$ with respect to a^2 is $27\pi^4/4$. Now, let $\sigma_r \geq 0$, we necessarily have from (31) that

$$\frac{27\pi^4}{4} < \frac{ga\chi}{v\beta}$$

(32)

Hence if

$$\frac{27\pi^4}{4} \geq \frac{ga\chi}{v\beta}$$

(33)

then $\sigma_r < 0$. Therefore, the system is stable.

Hence, The system is stable under condition (33) and also the system gets unstable under condition (32)

In the absence of rotation and magnetic field, (27) reduces to $\sigma_i\left[I_1 + \frac{ga\chi a^2}{v\beta}\left(\frac{G}{G-1}\right)p_1 I_3 + F I_{10}\right] = 0$ (34)

and the terms in brackets are positive definite. Thus, $\sigma_i = 0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is valid. The viscoelasticity, magnetic field and rotation introduce oscillatory modes (as σ_i may not be zero) in the system which were non–existent in their absence.

CONCLUSION

Combined effect of various parameters that is magnetic field, rotation and compressibility has been investigated on thermal instability of a walters'B' fluid. The main results from the analysis of the paper are:

i.   Stationary conversion, walters'B′ fluid behaves like an ordinary Newtonian fluid due to the vanishing of the viscoelastic parameter.
ii.   To study the effects of rotation and magnetic field, we examine the nature of $\frac{dR_1}{dQ_1}$ analytically. It has been found the rotation is stabilizing effect of the system, whereas the magnetic field has both stabilizing and destabilizing effects.
iii. The magnetic field rotation and viscoelasticity introduce oscillatory modes in the system which were non-existent in their absence.

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