International Journal of Scientific Engineering and Technology (ISSN : 2277-1581)
Volume 2 Issue 4, pp : 272-277 1 April 2013

Dynamic Modelling & Controller Design for Z-Source DC-DC Converter

Shilpa Sarode[1] S.G. Kadwane[2]
[1]Assistant Professor KITS, Ramtek, Nagpur, India

[2]Associate Professor, Y.C.C.E., Nagpur-India
[1]ashtankar.shilpa@ymail.com, [2] sgkadwane@gmail.com

Abstract— **This paper presents the detailed mathematical modeling of Z-source dc-dc converter (ZSC) in continuous conduction mode. Transfer function of ZSC is derived based on mathematical modeling with state space averaging method. This paper has been focused on dynamic modeling of open loop transfer function of ZSC along with design of closed loop controller. MATLAB based simulation results are presented for open loop and close loop system of ZSC.**

Keywords-component-* *Z-source dc-dc converter(ZSC), continuous conduction mode (CCM), Mathematical modelling, controller design.

I. INTRODUCTION

System involving power converters are being often used in applications like alternative energy sources and hybrid electric vehicle (HEV). Major objective for power electronics designers are efficiency, low cost & reliability. New topologies in power conversion like Z-source converter are invented to give better results in some applications [1]. ZSC is very promising new topology in power conditioning of alternative energy sources and applications like HEVs & utility interfacing [2]. ZSC can be implemented as a 3-phase dc/ac converter known as Z-source inverter (ZSI). It can also be applied to ac/dc & ac/ac power conversion. Unique buck- boost capability of ZSC allows a wider input voltage range & eliminates the usage of traditional converter [3-4]. Z-source has been recently studied and investigated by several researchers [1-8]. In ZSC the shoot-through state is allowed. In this state both upper & lower switches of the same phase leg are turned on. Shoot-through state is forbidden in traditional converter like VSI or CSI [6].

The Z-Source inverter(ZSI) has been introduced in order to overcome the limitations of traditional converter .The ZSI has unique buck-boost capability which ideally gives an output voltage range from zero to infinity regardless of the input voltage. The additional functionality of ZSI over the traditional inverter can be stated not only in terms of boost for DC to AC power conversion but a short circuit across any phase leg is allowed & dead band is not required. The second order filter is provided which is more efficient in suppressing output voltage ripples. The inrush current and harmonics can be reduced. Therefore the output distortion can be reduced and reliability is improved [7]. Most of the literature mentioned above are indeed focus on the applications and implementation but unfortunately does not discuss in depth the mathematical model and analysis of the controller design.

In this paper, operating principle of Z-source dc-dc converter is explained. Further the steady state model of ZSC is obtained for continuous conduction mode to study the dynamics introduced by inductors and capacitors uniquely contained in the circuit. The paper is organized as follows. The review of Z-source dc-dc converter is presented in section II. The state space representation and transfer function in CCM operation is derived further in section III. The design of closed loop controller along with design component of ZSC is given in section IV. Design oriented analysis is elaborated in section V. Simulation results are presented in section VI followed by summery of the work presented in last section VII.

II. REVIEW OF Z-SOURCE DC-DC CONVERTER

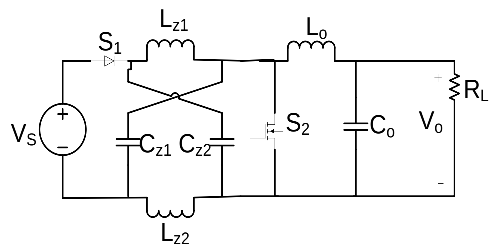


Figure1: Simulation & prototype system configuration of Z-source dc-dc converter

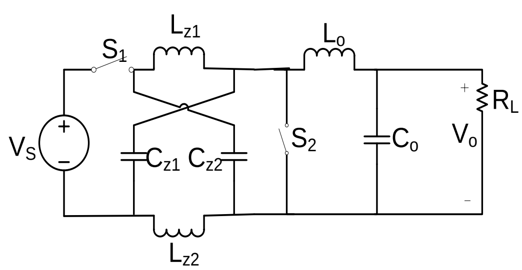
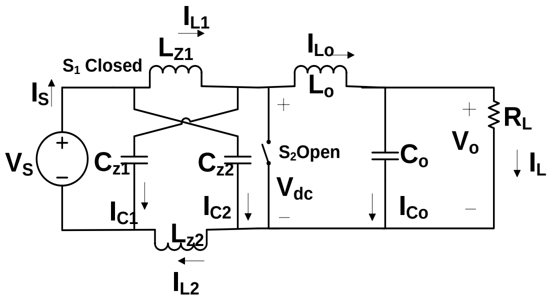


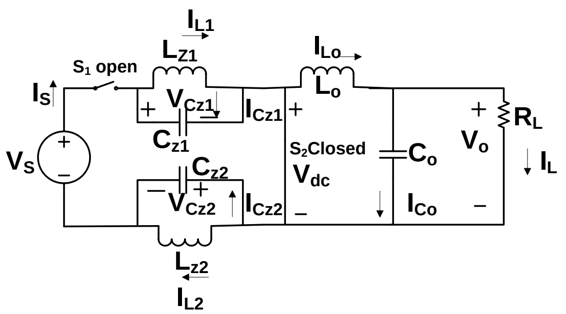
Figure2: Simplified circuit diagram of Z-source dc-dc converter

Figure 1 shows the general Z-Source converter structure, which consists of inductors (Lz_1 & Lz_2) connected in X shape to couple the inverter to the dc voltage source, which may be a battery, diode, rectifier or fuel cell. The Z-Source converter can produce a desired dc voltage regardless of dc source voltage. Figure 2 shows the simplified circuit diagram Z-source dc-dc converter where diode is replaced by switch S_1 & MOSFET switch is replaced by switch S_2.

The ZSC has two operating modes: Non shoot through mode and Shoot through mode. Figure 3 shows the equivalent circuit of ZSI at the non-shoot through mode and shoot through mode respectively [2-3].



(a)



(b)

Figure 3 Equivalent circuit of ZSC (a) Non-shoot through mode (b) Shoot through mode

In non shoot through mode as shown in Figure 3(a) switch S_2 is off in this mode where Z-source inductor L_Z, transfer the stored energies on them to load also the input current is transferred to Z- source capacitor C_Z and load. Inductor L_o is energized during this mode. In this mode as diode is forward biased switch S_1 is closed. In shoot-through mode as shown in Figure 3(b) switch S_2 is switched on. In this mode L_z are energized by C_Z. By applying Kirchhoff's voltage law to figure 3(b) voltage across diode (switch S_1) comes out to be negative value. Diode becomes reversed biased and hence switch S_1 becomes open .The load is meanwhile fed by filter inductor L_o and C_o.

The equation of the capacitor is derived in reference [7]. Assume the Z-source inductors (L_{Z1} & L_{Z2}) & capacitors (C_{Z1} & C_{Z2}) respectively. From the equivalent circuit, we have

$$V_{Lz1} = V_{Lz2} = V_{Lz}, \quad V_{Cz1} = V_{Cz2} = V_{Cz} \tag{1}$$

When the ZSC is in the non-shoot through state for a period T_1 from Figure 2(a) the inductor voltage and input voltage of the inverter can be expressed as

$$V_{Lz} = V_s - V_{Cz}, \quad V_{dc} = V_{Cz} - V_{Lz} = 2V_{Cz} - V_s \tag{2}$$

When the ZSC is in the shoot through state for a period T_o from Fig 2(b), the voltage Vdc becomes zero.The inductor voltage can be expressed as

$$V_{Lz} = V_{Cz} \tag{3}$$

As the average of the inductor voltage over one switching period T becomes zero in steady state, the capacitor voltage can be derived as

$$V_{Cz} = \frac{T_1}{T_1 - T_o} V_s = \frac{1-D}{1-2D} V_s \tag{4}$$

Where $T = T_1 + T_o$ is the switching period & $D = T_o / T$ is the shoot through time duty ratio.

Vcz is the steady state (dc) value of capacitor voltage & V_s is the steady state value of the input voltage. . Similarly output voltage Vo can be derived as

$$V_o = \frac{1-D}{1-2D} V_s \tag{5}$$

As equation (4) and (5) has equal right hand side hence left hand side should be same. Hence

$$V_o = V_{Cz} \tag{6}$$

The peak value (Vdcn) of the capacitor voltage is dependent on shoot through time & can be stepped up by increasing the shoot- through time. The peak value of the pulsating dc link voltage (Vdc) is given as

International Journal of Scientific Engineering and Technology (ISSN : 2277-1581)
Volume 2 Issue 4, pp : 272-277 1 April 2013

$$V_{dcn} = 2V_{cz} - V_s = \frac{T}{T_1 - T_o} V_s = \frac{1}{1-2D} V_s = BV_s$$

Where B is known as boosting factor on D.

III. STATE SPACE REPRESENTATION AND TRANSFER FUNCTION IN CCM OPERATION

The state variable Z-source capacitor voltage(Vcz), Z-source inductor current(ILz), output inductor current(ILo),output capacitor voltage(Vc_o) can be chosen as vector. $x(t) = [i_{Lz}(t) \quad V_{Cz}(t) \quad i_{Lo}(t) \quad V_{Co}(t)]$

The differential equation in shoot through mode can be written as in state form as

$$K\frac{dx(t)}{dt} = A_1 x(t) + B_1 u(t)$$

$$\begin{bmatrix} L_z & 0 & 0 & 0 \\ 0 & C_z & 0 & 0 \\ 0 & 0 & L_o & 0 \\ 0 & 0 & 0 & C_o \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_{Lz}(t) \\ V_{Cz}(t) \\ i_{Lo}(t) \\ V_{Co}(t) \end{bmatrix} = \cdot$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1/RL \end{bmatrix} \begin{bmatrix} i_{Lz}(t) \\ V_{Cz}(t) \\ i_{Lo}(t) \\ V_{Co}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} [V_s(t)] \qquad [7]$$

$$[K] = \begin{bmatrix} L_z & 0 & 0 & 0 \\ 0 & C_z & 0 & 0 \\ 0 & 0 & L_o & 0 \\ 0 & 0 & 0 & C_o \end{bmatrix}$$

$$[A_1] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1/RL \end{bmatrix} \quad [B_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Similarly, equation in non-shoot through mode can also be written in state space form as follows

$$K\frac{dx(t)}{dt} = A_2 x(t) + B_2 u(t)$$

$$\begin{bmatrix} L_z & 0 & 0 & 0 \\ 0 & C_z & 0 & 0 \\ 0 & 0 & L_o & 0 \\ 0 & 0 & 0 & C_o \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_{Lz}(t) \\ V_{Cz}(t) \\ i_{Lo}(t) \\ V_{Co}(t) \end{bmatrix} =$$

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 2 & 0 & -1 \\ 0 & 0 & 1 & -1/RL \end{bmatrix} \begin{bmatrix} i_{Lz}(t) \\ V_{Cz}(t) \\ i_{Lo}(t) \\ V_{Co}(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} [V_s(t)] \quad (8)$$

$$[A_2] = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 2 & 0 & -1 \\ 0 & 0 & 1 & -1/RL \end{bmatrix} \quad [B_2] = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

The duty ratio of S_1 is defined as (1-D). The small signal relationship among the state variable is derived by applying small signal perturbation to $\hat{V}_s(t)$ to input voltage & $\hat{d}(t)$ to shoot through duty ratio of S_2 shown b$V_s(t) = V_s + \hat{V}_s(t)$ & $d(t) = D + \hat{d}(t)$ [2],By combining equation (7) &(8), we obtain the small signal state equation as

$$K\frac{d\,x(t)}{dt} = (DA_1 + (1-D)A_2)\,\hat{x}(t) + (DB_1 + (1-D)B_2)\,\hat{u}(t) + \{(A_1 - A_2)\,X + (B_1 - B_2)U\}\,\hat{d}(t) \quad (9)$$

By taking Laplace transformation, equation (9) becomes as

$$sL_o\,\hat{i}_{Lo}(s) = (2-2D)\hat{V}_{Cz}(s) - \hat{V}_{Co}(s) + (D-1)\hat{V}_s(s) - (-2V_{Cz} - V_s)\,\hat{d}(s)$$

$$sL_z\,\hat{i}_{Lz}(s) = (2D-1)\hat{V}_{Cz}(s) + (1-D)\hat{V}_s(s) + (2Vc - V_s)\,\hat{d}(s)$$

$$sC_z\,\hat{V}_{Cz}(s) = (1-2D)\,\hat{i}_{Lz}(s) + (i_{Lo} - 2i_{Lz})\,\hat{d}(s) + (D-1)\,\hat{i}_{Lo}(s)$$

$$sC_o\,\hat{V}_{Co}(s) = \hat{i}_{Lo}(s) - \frac{\hat{V}_{Co}(s)}{R_L}$$

$$s\hat{i}_S(t) = (2-2D)\,\hat{i}_{Lz}(s) + (D-1)\,\hat{i}_{Lo}(s) + (i_{Lo} - 2i_{Lz})\,\hat{d}(s) \qquad (10)$$

From above equations the transfer function of ZSC is derived as duty factor to output voltage, $G_{vd}(s)$, as follows. The small signal expression for capacitor voltage can be expressed as

$$V_{Co}(s) = \hat{d}(s)G_{vd}(s) + \hat{V}_s(s)G_{vg}(s) \quad (11)$$

By putting $\hat{V}s(s) = 0$ control to output transfer function is given by

$$G_{vd}(s) = \frac{\hat{V}_{Co}(s)}{\hat{d}(s)}\Big|_{\hat{V}_s(s)=0} = \frac{\hat{V}_o(s)}{\hat{d}(s)}\Big|_{\hat{V}s(s)=0} \quad (12)$$

$$= \frac{\alpha_1 s^2 + \alpha_2 s + \alpha_3}{\beta_1 s^4 + \beta_2 s^3 + \beta_3 s^2 + \beta_4 s + \beta_5}$$

where

$$\alpha_1 = \frac{-L_z C_z V_s}{(1-2D)}, \quad \alpha_2 = \frac{(D-1)(2-2D)V_s L_z}{R_L(1-2D)^2}, \quad \alpha_3 = V_s$$

$$\beta_1 = L_z C_z L_o C_o, \quad \beta_2 = \frac{L_z L_o C_z}{R_L},$$

$$\beta_3 = L_o C_o (1-2D)^2 + L_z C_z + 2(1-D)^2 L_z C_o$$

$$\beta_4 = \frac{L_o}{R_L}(1-2D)^2 + \frac{2L_z}{R_L}(1-D)^2, \quad \beta_5 = (1-2D)^2$$

IV. DESIGN COMPONENTS OF ZSC & DESIGN OF CONTROLLER

In this section, computer simulation is conducted by using MATLAB programming. Parameters of ZSC used for computer simulation are shown in table below.

Parameter/component	Specifications
Input voltage, V_s	30V
Output voltage, V_o	40V
Output current, I_L	5A
Switching frequency, fs	20KHz
Load resistance, R_L	8.15Ω
Duty factor, D	0.2
Z-source inductor, L_z	300uH
Z-source capacitor, C_z	360uF
Output inductor, L_o	100uH
Output capacitor, C_o	500uF

Design of controller for ZSC is given as below

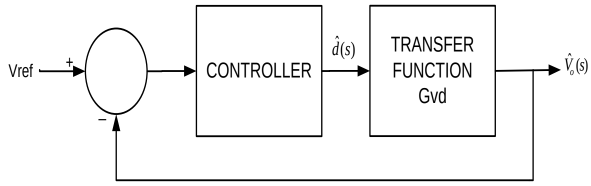


Figure 4: Block diagram of the controlled Z-Source dc-dc converter

Figure.4 shows main close loop block diagram for ZSC.The total gain around the closed loop gives the open loop transfer function, Gopen(s).It can be obtained as $Gopen(S) = T_{FC}(s).G_{vd}(s)$,Where T_{FC} (s) is the transfer function of controller.

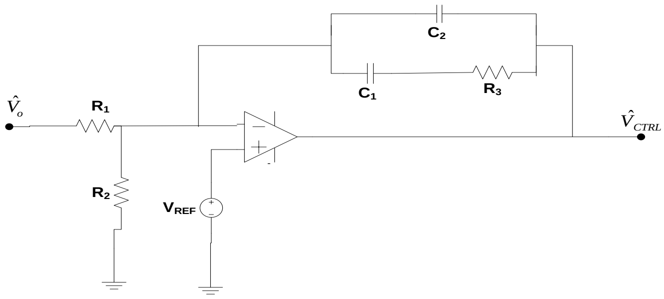


Figure5: PID controller of the Z-Source dc-dc converter

The controller is required in order to obtain non-oscillating & stable voltage at output. PID controller is used to keep the settling time and overshoot of the voltage minimum for changing the input voltage level. The open loop transfer function of PID controller $T_{FC}(S)$, is obtained as

$$T_{FC}(S) = \frac{R_1}{R_1 + R_2}\frac{1}{R_1 C_2}\frac{\{s + \frac{1}{C_1 R_3}\}}{s\{s + \frac{C_1 + C_2}{C_1 C_2 R_3}\}} \quad (18)$$

The circuit parameter used for PID controller is as follows

R_1	59KΩ
R_2	1000Ω
R_3	5110Ω
C_1	1000nF
C_2	500nF

VI.SIMULATION RESULT

The simulation result of Z-source dc-dc converter is as shown below. The simulation is carried out by using MATLAB. Figure 6 shows bode plot for compensated and uncompensated system for resistive load. Open loop and close loop simulation results of Z-source dc-dc converter plotted for output voltage(Vo), load current(IL),Capacitor voltage(Vc) and input voltage(Vs) .

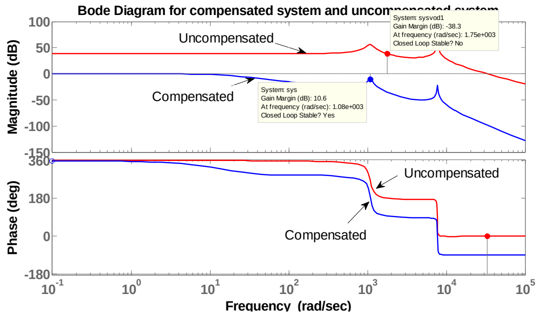


Figure 6: Bode plot (magnitude and phase) for compensated and uncompensated system for resistive load.

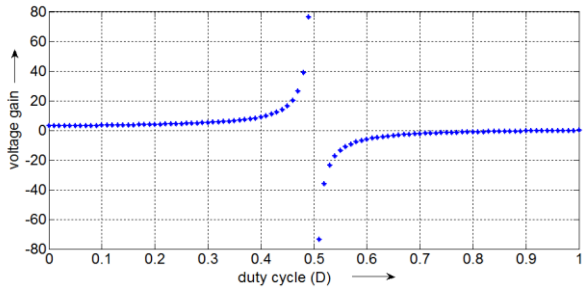


Figure7. voltage gain versus duty cycle of Z-source dc-dc converter

Figure 7 shows the plot for voltage gain vs duty cycle. It shows that ZSC can be operated in buck/boost mode depending upon the operating region of duty cycle. When D < 0.5, converter operates in boost mode & when D > 0.5, it operates in buck-boost mode [3].

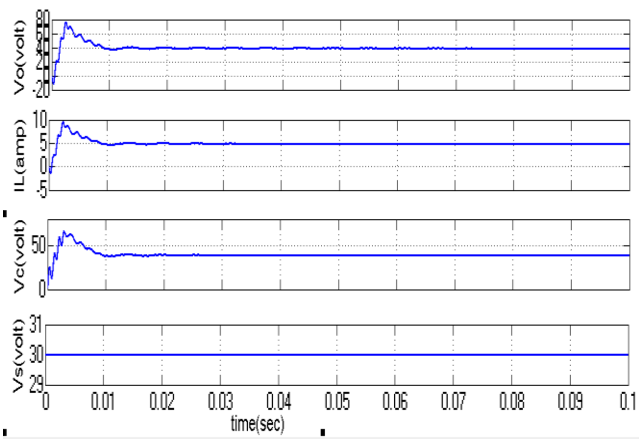


Figure 8.Simulation results of open loop system of Z-source dc-dc converter

While figure8 shows the simulation results of open loop system of ZSC.While figure 9 shows simulation results for open loop system for 50% step increase in input voltage Vs.

Figure 10 shows the simulation result of close loop system of ZSC while figure11 shows the simulation result of close loop system for step increase of 30% in input voltage Vs.

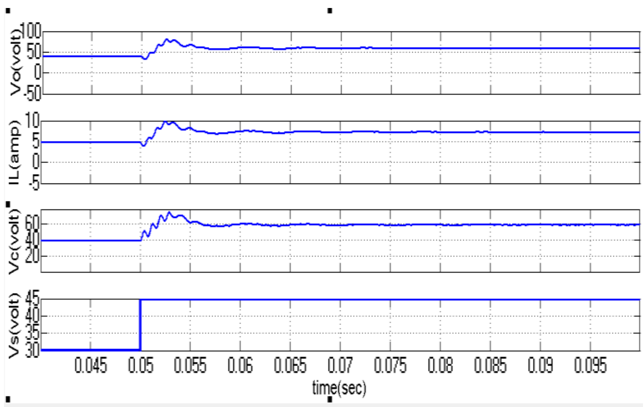


Figure 9. Simulation results of open loop system of Z-source dc-dc converter for 50% step increase in input voltage Vs.

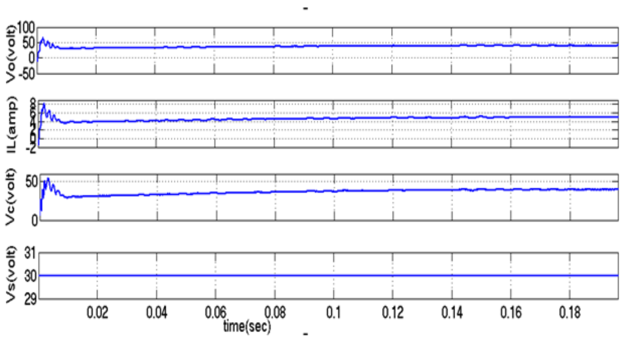


Figure10.Simulation result of close loop system of ZSC

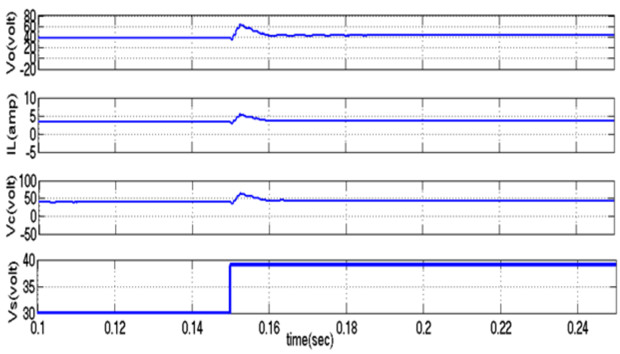


Figure 11. Simulation results of close loop system of Z-source dc-dc converter for 30% step increase in input voltage Vs

VII. CONCLUSION

In this paper, the mathematical modeling of ZSC using the method of state space averaging have been studied, similarly transfer function for ZSC have been established by using small signal circuit equation. This paper has focused on dynamic modeling of open loop transfer function along with close loop controller. Based on dynamic modeling and transfer function bode plot is obtained for compensated and uncompensated system. MATLAB based simulation results are obtained for open loop and close loop system with step increase in input voltage.

Acknowledgment

Authors acknowledge the work to the authorities of YCCE for their support.

REFERENCES

I. F.Z .Peng, "Z-source inverter," IEEE Trans. Ind.Appl., vol.39, no.2, pp.504–510, Mar./Apr.2003

II. J.liu,J.Hu and L.Xu, "Dynamic Modeling & Analysis Of Z-Source converter –Derivation of AC Small Signal Model and Design –Oriented Analysis",IEEE Transaction on Power Electronics vol.22, no.5, sept.2007, pp.1786-1796.

III. X. Fang, " A Novel Z-Source dc- dc converter", IEEE International Conference On Industrial Technology(ICIT) ,pp .1-4, Chengdu, China,21-24 April 2008.

IV. F.Z.Peng, M.Shen, and Z.Qian, "Maximum boost control of the Z-source inverter," IEEE Transaction Power Electron., vol.20, no.4, pp.833–838, Jul.2005.

V. Y.Xie,Z.Qian,X.Ding,F.Peng,"A Novel Buck-Boost Z-Source Rectifier",37th IEEE Power Electronics Specialists Conference, PESC,pp.1,Jeju,Korea,18-22 June 2006.

VI. V.P.Galigekere &M.K.Kazimierczuk,"Small Signal Modeling of PWM Z- Source Converter by Circuit Averaging Technique", in IEEE International Symposium on Circuits & System(ISCAS),May 2011,pp.1600-1603.

VII. Q.Tran, T.Chun, J.Ahn, and H.Lee, "Algorithms for controlling both the DC boost and AC output voltage of Z-source inverter," IEEE Trans. Ind.Electron.,vol.54,no.5,pp.2745–2750,Oct.2007.

VIII. Veda Prakash Galigekere and Marian K. Kazimierczuk, Fellow, IEEE, "Analysis of PWM Z-Source DC-DC Converter in CCM for Steady state",IEEE Transaction on Circuits and Systems-I:Regular Paper ,vol.59,no.4,pp.854-863,April-2012.

IX. N. Mohan, W.P.Robbin ,and T.Undeland, PowerElectronics :Con-verters,Applications,and Design, 2nd ed.NewYork:Wiley,1995

X. R.W.Ericson, D.Maksimovic, "Fundamentals of Power Electronics", Norwell,MA:Kluwer,2nd Edition,2001.

XI. S.Skogestad and I.Postleethwaite, Multivariable Feedback Control. New York: Wiley, 1996.